9. Observation Equations

9.1 Phase Pseudoranges

Let us briefly discuss the observation equations. Only the most important aspects are discussed here. For more information the reader is referred to e.g. [*Rothacher*, 1992], [*Mervart*, 1995]. Let us use the following notation:

t ... is the signal reception time (GPS time),
t_k ... is the reading of the receiver clock at the signal reception time,
δ_k ... is the error of the receiver clock at time t with respect to GPS time. The signal reception time t may be written as

$$t = t_k - \delta_k \ . \tag{9.1}$$

τ ... is the signal traveling time,
$\mathbf{r}_k(t)$ ... is the position of receiver k at signal reception time t,
$\mathbf{r}^i(t-\tau)$ ... is the position of the satellite i at signal emission time $t-\tau$ and
ϱ_k^i ... is the geometrical distance between satellite i (at signal emission time $t-\tau$) and receiver k (at signal reception time t).

The geometric distance ϱ_k^i may be written as

$$\varrho_k^i = c\,\tau \tag{9.2}$$

(c is the velocity of light) and at the same time as

$$\varrho_k^i = |\mathbf{r}_k(t) - \mathbf{r}^i(t-\tau)| \ . \tag{9.3}$$

Using the approximation

$$\mathbf{r}^i(t-\tau) = \mathbf{r}^i(t) - \dot{\mathbf{r}}^i(t)\ \tau \tag{9.4}$$

we obtain the following equation which may be solved for τ:

$$\begin{aligned}\Big(c^2 - \dot{\mathbf{r}}^i(t)\cdot\dot{\mathbf{r}}^i(t)\Big)\ \tau^2 - 2\,\dot{\mathbf{r}}^i(t)\ \Big(\mathbf{r}_k(t) - \mathbf{r}^i(t)\Big)\ \tau - \\ - \Big(\mathbf{r}_k(t)\cdot\mathbf{r}_k(t) - 2\,\mathbf{r}_k(t)\cdot\mathbf{r}^i(t) + \mathbf{r}^i(t)\cdot\mathbf{r}^i(t)\Big) = 0\end{aligned} \tag{9.5}$$

The GPS receiver measures the difference between two phases. The basic form of the observation equation may be written as follows

$$\psi_{Fk}^i(t) = \phi_{Fk}(t) - \phi_F^i(t-\tau) + n_{Fk}^i \ , \tag{9.6}$$

where

$\psi^i_{Fk}(t)$ ... is the phase measurement (in cycles) at epoch t and frequency F,
$\phi_{Fk}(t)$ ... is the phase generated by the receiver oscillator at signal reception time t,
$\phi^i_F(t-\tau)$ ... is the phase of the carrier at emission time $t-\tau$, and
n^i_{Fk} ... is the unknown integer number of cycles (the so-called *initial phase ambiguity*).

Using a Taylor series development we may rewrite the last equation as

$$\psi^i_{Fk}(t) = \phi_{Fk}(t) - \phi^i_F(t) + \tau\, f_F + n^i_{Fk} \;, \tag{9.7}$$

where f_F is the frequency of the carrier. The difference

$$\phi_{Fk}(t) - \phi^i_F(t)$$

is zero in the case of ideal oscillators and is equal to

$$(\delta_k - \delta^i)\, f_F$$

if the receiver clock error δ_k and the satellite clock error δ^i are taken into account. The observation equation is then given by

$$\psi^i_{Fk}(t) = (\delta_k - \delta^i)\, f_F + \tau\, f_F + n^i_{Fk} \;. \tag{9.8}$$

Multiplying this equation by the wavelength λ_F we receive

$$L^i_{Fk} = \varrho^i_k + c\,\delta_k - c\,\delta^i + \lambda_f\, n^i_{Fk} \;. \tag{9.9}$$

9.2 Code Pseudoranges

Using the known codes modulated onto the GPS carriers, the GPS receivers are able to measure the quantity

$$P^i_k = c\,((t+\delta_k) - (t-\tau+\delta^i)) \;, \tag{9.10}$$

which is called *pseudorange* (because of the biases caused by satellite and receiver clock errors. Using the geometrical distance ϱ^i_k the equation may be written as

$$P^i_{Fk} = \varrho^i_k + c\,\delta_k - c\,\delta^i \;. \tag{9.11}$$

9.3 Receiver Clocks

We will see in the section 9.5 that the term $c\,\delta_k$ in eqns. (9.9) and (9.11) may be eliminated by forming the differences of the measurements to two satellites (the term $c\,\delta^i$ may be eliminated using the differences between two receivers). This does not mean, however, that in the differences the receiver clock error δ_k is *completely* eliminated. By looking at eqns. (9.1) and (9.3) it becomes clear, that to compute the geometric distance between the satellite and the receiver at time t (in GPS time scale) the receiver clock error δ_k has to be known to correct the reading of the receiver clock t_k

$$\varrho^i_k(t) = \varrho^i_k(t_k - \delta_k) \;. \tag{9.12}$$

By taking the time derivative of thus equation we obtain

$$\mathrm{d}\,\varrho_k^i = -\dot{\varrho}_k^i\,\mathrm{d}\,\delta_k\ , \tag{9.13}$$

where $\dot{\varrho}_k^i$ is the radial velocity of the satellite with respect to the receiver. This velocity is zero if the satellite is at the point of closest approach and may reach values up to 900 $\mathrm{m}\cdot\mathrm{s}^{-1}$ for zenith distances $z \approx 80^{\circ}$. $\mathrm{d}\,\varrho_k^i$ may be interpreted as the error in the distance ϱ_k^i we make, when assuming an error $-\mathrm{d}\,\delta_k$ in the receiver clock synchronization with the GPS time. We conclude that the error $|\mathrm{d}\,\varrho_k^i|$ in the geometric distance ϱ_k^i induced by a receiver clock error $|\mathrm{d}\,\delta_k|$ will be smaller than 1 mm if the receiver clock error $|\mathrm{d}\,\delta_k|$ is smaller than 1 μs.

9.4 Measurement Biases

The phase measurements and the code pseudoranges are affected by both, systematic errors and random errors. There are many sources of the systematic errors (satellite orbits, clocks, propagation medium, receiver clocks, relativistic effects, antenna phase center variations, etc.) In the Bernese GPS Software all relevant systematic errors are carefully modeled. Here we discuss only two kinds of systematic errors, namely tropospheric and ionospheric refraction.

$\Delta\varrho_k^i$ ... is the so-called *tropospheric refraction*. It is the effect of the neutral (i.e. the non-ionized) part of the earth's atmosphere. It is important that tropospheric refraction does *not* depend on the frequency and that the effect is the same for phase measurements and code measurements.

I_k^i ... is the so-called *ionospheric refraction*. The ionosphere is a dispersive medium for microwave signals, which means that the refractive index for GPS signals is frequency-dependent.

In a first (but excellent) approximation ionospheric refraction is proportional to

$$\frac{1}{f^2}\ ,$$

where f is the carrier frequency. In our notation the term I_k^i is the effect of the ionosphere on the first carrier L_1. The ionospheric refraction on the second carrier L_2 will be

$$\frac{f_1^2}{f_2^2}\, I_k^i\ .$$

Ionospheric refraction delays the GPS code measurements and advances the carrier phases. The effect has the same absolute value for code and phase measurements, but the signs are opposite.

Taking into account tropospheric refraction and ionospheric refraction we may rewrite the observation equations (9.9) and (9.11) for both frequencies and both types of measurements (phase and code). We use the same notation for the geometrical distance ϱ_k^i although eqns. (9.9) and (9.11) implicitly contain tropospheric and ionospheric delays. Eqns. (9.14) are most refined version of the observation equations (9.9) and (9.11).

$$
\begin{aligned}
L^i_{1k} &= \varrho^i_k - I^i_k + \Delta\varrho^i_k + c\,\delta_k - c\,\delta^i + \lambda_1\, n^i_{1k} && \text{(9.14a)}\\
L^i_{2k} &= \varrho^i_k - \frac{f_1^2}{f_2^2}\, I^i_k + \Delta\varrho^i_k + c\,\delta_k - c\,\delta^i + \lambda_2\, n^i_{2k} && \text{(9.14b)}\\
P^i_{1k} &= \varrho^i_k + I^i_k + \Delta\varrho^i_k + c\,\delta_k - c\,\delta^i && \text{(9.14c)}\\
P^i_{2k} &= \varrho^i_k + \frac{f_1^2}{f_2^2}\, I^i_k + \Delta\varrho^i_k + c\,\delta_k - c\,\delta^i && \text{(9.14d)}
\end{aligned}
$$

9.5 Forming Differences

Differences of the original observations allow it to eliminate or reduce some biases. Let us define the *single difference* (between a pair of receivers) by

$$L^i_{Fk\ell} = L^i_{Fk} - L^i_{F\ell} \tag{9.15}$$

and the *double difference* (between a pair of receivers and between a pair of satellites) by

$$L^{ij}_{Fk\ell} = L^i_{Fk\ell} - L^j_{Fk\ell}\,. \tag{9.16}$$

The double differences are the basic observables in the Bernese GPS Software. The corresponding observation equations are

$$
\begin{aligned}
L^{ij}_{1k\ell} &= \varrho^{ij}_{k\ell} - I^{ij}_{k\ell} + \Delta\varrho^{ij}_{k\ell} + \lambda_1\, n^{ij}_{1k\ell} && \text{(9.17a)}\\
L^{ij}_{2k\ell} &= \varrho^{ij}_{k\ell} - \frac{f_1^2}{f_2^2}\, I^{ij}_{k\ell} + \Delta\varrho^{ij}_{k\ell} + \lambda_2\, n^{ij}_{2k\ell} && \text{(9.17b)}\\
P^{ij}_{1k\ell} &= \varrho^{ij}_{k\ell} + I^{ij}_{k\ell} + \Delta\varrho^{ij}_{k\ell} && \text{(9.17c)}\\
P^{ij}_{2k\ell} &= \varrho^{ij}_{k\ell} + \frac{f_1^2}{f_2^2}\, I^{ij}_{k\ell} + \Delta\varrho^{ij}_{k\ell} && \text{(9.17d)}
\end{aligned}
$$

By forming the double difference observations the receiver clock errors and the satellite clock errors are eliminated (assuming that the receiver clock errors are known accurately enough to compute the distances ϱ correctly – see section 9.3).

Using double difference observations from two different epochs t_1 and t_2 the *triple difference* may be formed. In the Bernese GPS Software the triple differences of the phase measurements are used in the data pre-processing.

$$
\begin{aligned}
L^{ij}_{1k\ell}(t_2) - L^{ij}_{1k\ell}(t_1) &= \varrho^{ij}_{k\ell}(t_2) - \varrho^{ij}_{k\ell}(t_1) - \Big(I^{ij}_{k\ell}(t_2) - I^{ij}_{k\ell}(t_1)\Big) && \text{(9.18a)}\\
L^{ij}_{2k\ell}(t_2) - L^{ij}_{2k\ell}(t_1) &= \varrho^{ij}_{k\ell}(t_2) - \varrho^{ij}_{k\ell}(t_1) - \frac{f_1^2}{f_2^2}\Big(I^{ij}_{k\ell}(t_2) - I^{ij}_{k\ell}(t_1)\Big) && \text{(9.18b)}
\end{aligned}
$$

In above equations we assumed that the unknown ambiguity parameters $n^{ij}_{1k\ell}, n^{ij}_{2k\ell}$ remained the same within the time interval $<t_1, t_2>$ and that therefore the phase ambiguities are eliminated (the main advantage of the triple differences). This is indeed true if the receivers did not loose lock within this time interval and if no cycle slip occurred. Tropospheric refraction usually does not change rapidly in time and therefore it is considerably reduced on the triple difference level. This is not true, however, for the ionospheric refraction which may show very rapid variations in time in particular in high northern and southern latitudes.

9.6 Linear Combinations of Observations

Often it is useful to form linear combinations of the original carrier phase and/or code measurements. The linear combinations used in the Bernese GPS Software are discussed in this section. We form the linear combinations using either zero or double difference measurements. L_1, L_2 represent the phase observables (zero or double differences), P_1, P_2 represent the code observables, both in units of meters.

The Ionosphere-free Linear Combination L_3

The linear combination

$$L_3 = \frac{1}{f_1^2 - f_2^2}(f_1^2\, L_1 - f_2^2\, L_2) \tag{9.19}$$

is often called "ionosphere-free" because the ionospheric path delay is practically eliminated. The same is true for the corresponding combination of code measurements

$$P_3 = \frac{1}{f_1^2 - f_2^2}(f_1^2\, P_1 - f_2^2\, P_2)\ . \tag{9.20}$$

Taking into account the double difference phase measurements and neglecting tropospheric refraction $\Delta\varrho_{k\ell}^{ij}$ in eqns. (9.17a) and (9.17b) the ionosphere-free linear combination has the form

$$L_{3k\ell}^{ij} = \varrho_{k\ell}^{ij} + B_{3k\ell}^{ij}\ , \tag{9.21}$$

where the ionosphere-free bias $B_{3k\ell}^{ij}$ may be written as

$$B_{3k\ell}^{ij} = \frac{1}{f_1^2 - f_2^2}\left(f_1^2\lambda_1 n_{1k\ell}^{ij} - f_2^2\lambda_2 n_{2k\ell}^{ij}\right)\ . \tag{9.22}$$

This bias cannot be expressed in the form $\lambda_3\, n_{3k\ell}^{ij}$, where $n_{3k\ell}^{ij}$ is an integer ambiguity. If we know the difference $n_{5k\ell}^{ij} = n_{1k\ell}^{ij} - n_{2k\ell}^{ij}$ (the so-called wide-lane ambiguity — see below), however, the ionosphere-free bias $B_{3k\ell}^{ij}$ may be written as

$$B_{3k\ell}^{ij} = c\,\frac{f_2}{f_1^2 - f_2^2}\, n_{5k\ell}^{ij} + \underbrace{\frac{c}{f_1 + f_2}}_{\lambda_3}\, n_{1k\ell}^{ij}\ , \tag{9.23}$$

where the first term on the right-hand side is known. The formal wavelength λ_3 is only about 11 cm. Therefore the unknown ambiguity $n_{1k\ell}^{ij}$ in equation (9.23) is often called *narrow-lane* ambiguity.

The Geometry-free Linear Combination L_4

The linear combination

$$L_4 = L_1 - L_2 \tag{9.24}$$

is independent of receiver clocks and geometry (orbits, station coordinates). It contains the ionospheric delay and the initial phase ambiguities. It may be used for the estimation of ionosphere models. The same linear combination may be formed using the code observations, too.

The Wide-lane Linear Combination L_5

The linear combination

$$L_5 = \frac{1}{f_1 - f_2}(f_1\ L_1 - f_2\ L_2) \tag{9.25}$$

is used in the Bernese GPS Software on the double difference level for phase observations for the purpose of cycle slip fixing and ambiguity resolution. Using eqns. (9.17a) and (9.17b) and neglecting both, the ionospheric refraction $I^{ij}_{k\ell}$ and the tropospheric refraction $\Delta\varrho^{ij}_{k\ell}$, we obtain

$$L^{ij}_{5k\ell} = \varrho^{ij}_{k\ell} + \underbrace{\frac{c}{f_1 - f_2}}_{\lambda_5}\ \underbrace{(n^{ij}_{1k\ell} - n^{ij}_{2k\ell})}_{n^{ij}_{5k\ell}}\ . \tag{9.26}$$

The formal wavelength λ_5 is about 86 cm and is roughly 4 times longer than λ_1 or λ_2. Therefore this linear combination is called the *wide-lane* and the ambiguity

$$n^{ij}_{5k\ell} = n^{ij}_{1k\ell} - n^{ij}_{2k\ell} \tag{9.27}$$

is called the *wide-lane ambiguity*.

The Melbourne-Wübbena Linear Combination L_6

The Melbourne-Wübbena combination is a linear combination of both, carrier phase (L_1 and L_2) and P–code (P_1 and P_2) observables described by [*Wübbena*, 1985] and [*Melbourne*, 1985]. This combination eliminates the effect of the ionosphere, of the geometry, of the clocks, and of the troposphere. The combination is given by

$$L_6 = \frac{1}{f_1 - f_2}(f_1\ L_1 - f_2\ L_2) - \frac{1}{f_1 + f_2}(f_1\ P_1 + f_2\ P_2)\ . \tag{9.28}$$

For double difference observations we obtain

$$L^{ij}_{6k\ell} = \lambda_5\ n^{ij}_{5k\ell}\ . \tag{9.29}$$

With good P–code data (rms $\leq$ 1 m) this linear combination may be used for the resolution of the wide-lane ambiguities $n^{ij}_{5k\ell}$. On the zero difference level the same linear combination gives

$$L^{i}_{6k} = \lambda_5\ n^{i}_{5k} \tag{9.30}$$

which means that this linear combination may be used to check zero difference observations for cycle-slips. However, only the difference $n^i_{1k} - n^i_{2k}$ can be checked in this way.

The most important linear combinations together and characteristics are summarized in Table 9.1.

Carrier	Description	Wavelength	Noise rel to L_1	Ionosphere rel to L_1
L_1	Actual Carrier	19 cm	1	1
L_2	Actual Carrier	24 cm	1	1.6
L_3	Ionosphere-free LC	0 cm	3	0
L_4	Geometry-free LC	∞	1.4	0.6
L_5	Wide Lane	86 cm	5	1.3
L_6	Melbourne-Wüebbena	86 cm		0

Table 9.1: Linear Combinations of the L_1 and L_2 Observables used in the Bernese GPS Software Version 4

10. Data Pre-Processing

10.1 Overview

The first group of processing programs of the Bernese GPS Software is discussed here. The programs of this group do not produce final results but check and prepare the data for the main estimation program (GPSEST).

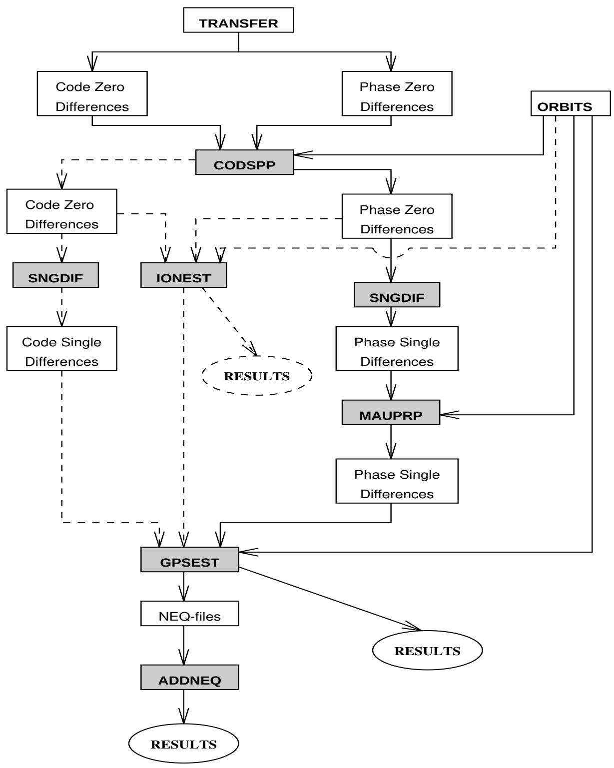


Figure 10.1: Functional Flow Diagram of the Processing Part

The simplified flow diagram for the entire processing part is given in Figure 10.1. The solid lines show the normal (and mandatory) procedure. The pre-processing programs used in this procedure are CODSPP, SNGDIF and MAUPRP. There are several other pre-processing programs in the software, however. Some of them are older programs, the others are service programs or programs used for special purposes.

10.2 Pre-Processing on the RINEX Level – Menu 2.7.5

RNXCYC is a pre-processing program using RINEX files pertaining to one or more stations. It looks for wide-lane cycle slips in the Melbourne-Wübbena linear combination on the zero difference level. and marks observations where a cycle slip was found with a cycle slip flag. Normally we do not use this program. It might be useful if one wants to use the zero difference phase measurements (e.g. in the program IONEST). Our standard screening program MAUPRP checks double differences. RNXCYC could *not* replace MAUPRP, so far. The method used in RNXCYC cannot detect a cycle slips happening between epochs t_1 and t_2 if equation

$$n^i_{1k}(t_2) - n^i_{1k}(t_1) = n^i_{2k}(t_2) - n^i_{2k}(t_1) \tag{10.1}$$

holds. RNXCYC only works with dual band P-code measurements, i.e. the receiver has to give L_1 and L_2 phase as well as P_1 and P_2 code observations (the Melbourne-Wübbena linear combination is used). The quality of the code measurements is critical.

10.3 Pre-Processing of Code Observations

10.3.1 Simple Non-Parametric Screening (CODCHK)

The program CODCHK (Menu 4.1) checks zero or single difference code observations for outliers. Usually it is not necessary to utilize this program in Version 4.0 . Outlier detection in zero difference code observations (originally the main purpose of CODCHK) has been implemented into the program CODSPP (Section 10.3.2). The algorithm used in CODCHK is the same as that in the first part of MAUPRP.

Algorithm

It is known that code (and code difference) observations are values of "smooth" time functions with random errors of the order of a few meters. The program checks whether or not the $q + 2$ subsequent observations may be represented within an interval of a few minutes by a low degree polynomial of degree q. This is done by computing the $(q + 1)$–st derivative of the observation time series and by checking whether or not this quantity is zero (within 3 times its rms error). The rms error of the $(q + 1)$–st derivative is computed from the rms error of the observations which is an input variable of the program. If the assumption is correct the interval considered is shifted by one observation, if not, the last observation of the current interval is marked and replaced by the following one. If the current interval gets longer than a maximum length specified by the user, all observations of the current interval are dropped, and the process is re-initialized. After successful re-initialization the program tests backwards to recover erroneously marked observations. The initialization works as follows: the assumption is tested using the first $q + 2$ observations (for re-initialization the next q+2 observations not yet checked). If it is wrong, the first observation is marked, the assumption is tested using observations $2, 3, \ldots, q + 3$. The process is terminated as soon as the above assumption is correct.

10.3.2 Single Point Positioning and Receiver Clock Synchronization (CODSPP)

In Section 9.3 we have seen that the receiver clock has to be synchronized with GPS time. The receiver clock error δ_k has to be known with an accuracy better than $1\ \mu$s. It would be possible to introduce δ_k as unknown parameters during the final least-squares adjustment in program GPSEST but this would increase the number of parameters considerably. Fortunately it is possible to compute δ_k a priori with sufficient ($1\ \mu$s) accuracy using the zero difference code measurements. This is the main task of program CODSPP (the second important result from this program are the receiver coordinates). Looking at eqn. (9.11) we conclude that if we want to reach an accuracy of $1\ \mu$s in δ_k it is necessary to have available the code measurements with an rms error smaller than

$$c\ (\delta_k)_{max} = c\ (1\ \mu\mathrm{s}) \approx 300\ \mathrm{m}$$

(c is the velocity of light). Obviously even C/A–code measurements are sufficient for this purpose. Of course CODSPP will process P-code, if available.

Program CODSPP uses the standard least-squares adjustment to compute the unknown parameters. The observables are the zero difference code measurements. Usually the L_3 (ionosphere-free) linear combination is used. The most important parameters computed by CODSPP are the receiver clock corrections δ_k. These parameters will be estimated in any case. The user may also estimate the coordinates of the receivers. The model used in the program is that represented by eqns. (9.14c,d). The unknown parameter δ_k appears implicitly in the term ϱ_k^i, too. Therefore CODSPP estimates the parameters iteratively (using a least-squares adjustment). The second reason for the iterations is, that the a priori coordinates may not be accurate enough.

The detection of outliers (see Panel 4.2–2) is a new feature of CODSPP in Version 4.0 . It allows to skip CODCHK.

10.4 Forming Baselines (SNGDIF) – Menu 4.3

The Bernese GPS Software uses double differences as basic observables. The single (between receivers) differences (see Section 9.5) are stored in files, the double differences are created in program GPSEST. Program SNGDIF creates the single differences and stores them in files. The program may create both, phase and code single differences. Usually only the phase single differences are used for further computations. An important exception is the ambiguity resolution using the Melbourne–Wübbena linear combination. In that case the code single differences have to be formed, too.

Strategies Used for Baseline Definition

Let us assume that N receivers are used simultaneously. Let us further assume that the same satellites are tracked by all receivers (this assumptions is true in local campaigns). We have thus N zero difference measurements to each satellite at each epoch (and each carrier). If we use single difference observations, only $N-1$ independent single differences may be formed.

If the assumption that the same set of satellites is tracked by all receivers is *not* correct (global campaigns) it would be better to optimize the forming of single differences for each epoch. However, the data handling would be tremendous in such case. We use a compromise in the Bernese GPS Software. We create only one set of $N-1$ baselines for the entire session (and store the observations into the single difference files – each single difference file corresponding to one baseline and one session), but we *optimize* the selection of these independent baselines. The algorithm used is known as maximum (or minimum) path method. First, the baselines are ordered according to a user-defined criterion (either baseline length or the number of available single difference observables – see below). Then, all the receivers active in the current session are given the initial flag 0. We take the "best" baseline into the optimal set, the two corresponding stations receive the flags 1. The variable "maximum flag" is set to 1. Now, we proceed to the second baseline. If the corresponding stations have flag 0 we change them to 2, and 2 is the value of the "maximum flag", too. If one station has flag 0 and the other 1, both flags will be set to 1 and the "maximum flag" remains 1. From now on we proceed as follows: we select the next baseline according to our criterion and make the distinction of the following four cases:

1) Both stations of the new baseline have the flags 0: In this case the two station flags are set to "maximum flag $+1$", and we have to increment the "maximum flag" accordingly.

2) One station has flag 0, the flag of the other station is not equal to 0: In this case the station with flag 0 receives the (non-zero) flag of the other station. The "maximum flag" is not changed.

3) The two flags are not equal and no flag is 0 : Let us assume that the first station has a lower flag than the second one. We have to change the flags of all stations which have the same flag as the first station. The station flags are set to the flag of the second station.

4) The two flags are equal and different from 0: It means that this baseline is dependent and cannot be added to the set.

This procedure is repeated until $N-1$ independent baselines have been formed. Usually we use the number of observations as optimization criterion. The other possibility is to use the baseline length as a criterion and to create the set of shortest baselines. This could be useful if you want to create the same set of baselines each session (assuming, that the same stations are observing each session). Baseline length is an important characteristic for ambiguity resolution. If the number of observations is used as a criterion, the program will not create very long baselines, either.

10.5 Pre-Processing Phase Observations

It was stated in Chapter 9 that the receivers can measure the difference between the phase of the satellite transmitted carrier and the phase of the receiver generated replica of the signal. This measurement yields a value between 0 and 1 cycle (0 and 2π). After turning on the receiver an integer counter is initialized. During tracking the counter is incremented by one whenever the fractional phase changes from 2π to 0. Thus for every epoch the accumulated phase is the sum of the directly measured fractional phase and the integer count. The initial integer number n^i_{Fk} of cycles between the satellite i and receiver k is unknown and has to be estimated (see eqns. (9.14)). This initial phase ambiguity remains the same as long as no loss of signal lock occurs. A loss of lock causes a jump in the instantaneous accumulated phase by an integer number of cycles. If there is a difference

$$n^i_{Fk}(t_2) - n^i_{Fk}(t_1) \neq 0 \tag{10.2}$$

we say that a cycle slip occurred between time t_1 and t_2. There are several possible sources for cycle slips:

- obstruction of the satellite signal due to trees, buildings, etc.,
- low signal-to-noise ratio due to rapidly changing ionospheric conditions, multipath, high receiver dynamics, or low satellite elevation,
- failure in the receiver software, and
- malfunctioning of the satellite oscillator.

The following tasks have to be accomplished during pre-processing:

1) Check all the observations and find the time intervals $< t_1, t_2 >$ which are corrupted by cycle slips.

2) If possible repair the cycle slips. We thus have to estimate the difference $n^i_{Fk}(t_2) - n^i_{Fk}(t_1)$ and to correct all observations following the epoch t_1 by this difference. If it is not possible to estimate this difference in a reliable way, the observation at time t_2 has to be marked as outlier or a new unknown ambiguity parameter $n^i_{Fk}(t_2)$ *must* be introduced.

There are three pre-processing programs in the Bernese GPS Software dealing with the tasks above. The first one is the program RNXCYC (see Section 10.2). We do not use it in general. The second program is called OBSTST (Menu 4.4.1). OBSTST is the predecessor of the pre-processing program we currently use. In version 4.0 of the Bernese GPS Software the principal pre-processing program is MAUPRP (Manual and AUtomatic PRe-Processing). It screens *single difference observation* files forming and analyzing all useful linear combinations of phase observations. The program either assumes that the wide-lane combination is not corrupted by cycle slips (this is true if the pre-processing program RNXCYC was used) or it looks for wide-lane cycle slips, too. The quality of results seems to be similar in both cases. MAUPRP does *not* use code measurements, the pre-processing is thus code-independent. This aspect is e.g. important when processing A/S data (the quality of the code measurements may be much lower under A/S). The pre-processing program MAUPRP consists of the following steps:

1. Checking by smoothing: The goal is to identify time intervals within which with utmost certainty there are no cycle slips. Usually a fair amount of such data (not corrupted by cycle slips) may be found. The program uses the same algorithm as program CODCHK (Section 10.3.1). It checks whether the double difference phase observations are values of a smooth function of time and whether they may be represented within an interval of a few minutes by a polynomial of low degree, say q, by computing the $(q + 1)$-st derivative and by checking whether or not this quantity is zero within the expected rms error.
2. Triple difference solution: With those data identified as "clean" in the first step a triple difference (Section 9.5) solution is performed using the standard least-squares adjustment for each baseline (the coordinates of the first receiver are kept fix on their a priori values, the coordinates of the second receiver are estimated). This solution is not as accurate as the result of the least-squares adjustment using double differences, but it is a fair approximation of the final solution. The main advantage over a double difference solution has to be seen in the fact that an undetected cycle slip corrupts one triple difference only (and not all double differences after the slip). The triple difference residuals are computed and stored in a scratch file (the residuals are computed for *all* observations not only for those identified as "clean" in the first step).
3. Automatic cycle slip detection: First, the program corrects big jumps on the single difference level. Such jumps usually originate from the receiver clock and are common to all satellites.

Therefore these *clock jumps* are irrelevant for double difference processing algorithms. Then the results of the previous two steps are used to detect the cycle slips in the following way:

The triple difference residuals stemming from the second step (they have been stored in an auxiliary file – see above) are inspected. We assume that we have observations in two carriers L_1 and L_2 and write

r_1 ... the triple difference L_1-residuum (we do not explicitly indicate the two receivers, two satellites, and two epochs pertaining to this triple difference) and

r_2 ... the triple difference L_2-residuum.

The user may select either `COMBINED` or `BOTH` method in Panel 4.4.2–1 (see the corresponding help panel). If the `COMBINED` method is used, MAUPRP interprets the residuals as follows:

$$r_1 = b_1\lambda_1 + \left(I^{ij}_{k\ell}(t_2) - I^{ij}_{k\ell}(t_1)\right) , \quad r_2 = b_2\lambda_2 + \frac{f_1^2}{f_2^2}\left(I^{ij}_{k\ell}(t_2) - I^{ij}_{k\ell}(t_1)\right) \tag{10.3}$$

where $I^{ij}_{k\ell}(t)$ is the ionosphere refraction "as seen" by the L_1 carrier at time t (see eqns. (9.17)). Now, we check whether the *no-cycle-slip hypothesis* $b_1 = b_2 = 0$ holds. The residual in L_3 (ionosphere-free) linear combination is computed as

$$r_3 = \beta_1\, r_1 + \beta_2\, r_2 \ , \ \text{where} \ \ \beta_1 = \frac{f_1^2}{f_1^2 - f_2^2} \ \ \text{and} \ \ \beta_2 = -\frac{f_2^2}{f_1^2 - f_2^2} \ . \tag{10.4}$$

The following condition should be met:

$$|r_3| \le 3\sqrt{8}\sqrt{(\beta_1\sigma_1)^2 + (\beta_2\sigma_1)^2} \tag{10.5}$$

(the factor $\sqrt{8} = \sqrt{2^3}$ is due to triple differencing). Eqns. (10.3) allow us to compute the change of ionospheric refraction between the epochs t_1 and t_2

$$I^{ij}_{k\ell}(t_2) - I^{ij}_{k\ell}(t_1)$$

independently from both carriers (we assume $b_1 = b_2 = 0$ at present). The mean value m is computed as

$$m = \frac{1}{2}\left(r_1 + \frac{f_2^2}{f_1^2} r_2\right) \tag{10.6}$$

We check whether the condition

$$m \le M_{ion} \tag{10.7}$$

is met. The value of M_{ion} and the a priori rms errors of the zero difference observables σ_1 and σ_2 are input variables (see Panel 4.4.2–3 and Panel 4.4.2–4). If conditions (10.5) and (10.7) hold, *the no-cycle-slip hypothesis is accepted* as true. In the opposite case a search over the values b_1 and b_5 is performed. All combinations

$$\begin{aligned} b_{1p} &= \text{NINT}\left(\frac{r_1}{\lambda_1}\right) + p \ , & p &= -J_1, \ldots, -1, 0, 1, \ldots, J_1 \\ b_{5q} &= \text{NINT}\left(\frac{r_1}{\lambda_1} - \frac{r_2}{\lambda_2}\right) + q \ , & q &= -J_5, \ldots, -1, 0, 1, \ldots, J_5 \end{aligned} \tag{10.8}$$

(NINT = nearest integer) are formed and the "corrected" residuals

$$r_{1p} = r_1 - b_{1p}\lambda_1 \ , \quad r_{2pq} = r_2 - (b_{1p} - b_{5q})\lambda_2 \tag{10.9}$$

are tested in the same way as the original residuals r_1 and r_2. The program user has to specify the search ranges J_1 and J_5 (see Panel 4.4.2–3). If one combination of r_{1p}, r_{2pq} meets the no-cycle-slip hypothesis, the observations are corrected by $b_{1p}\lambda_1$ or by $b_{2pq}\lambda_2$. If no "good" combination is found, a new ambiguity parameter should be introduced. But introducing too many ambiguity parameters would result in large rms errors of the other parameters estimated in GPSEST. There is still the chance that the problem is actually no cycle slip but an outlier and that only one or a few observations are corrupted. If the cycle slip problem appears in the triple difference between the epochs t_1 and t_2 the first corrective action is usually (see options in Panel 4.4.2–4) to mark (i.e. not use) the observation stemming from epoch t_2 and to try the same tests using the triple difference between the epochs t_1 and t_3 and possibly t_1 and t_4 etc. Of course, there has to be a parameter which limits the length of the interval $< t_1, t_x >$.

If the method `L1, L2,` or `BOTH` is selected in Panel 4.4.2–1, MAUPRP does not create any linear combination of the measurements. The value m is computed as

$$m = r_1 \quad \text{or} \quad m = \frac{f_2^2}{f_1^2}\, r_2 \tag{10.10}$$

and only the condition (10.7) is tested (and *not* the condition (10.5)).

Example 1

The first example stems from the pre-processing of the baseline Kootwijk–Wettzell (see Chapter 4). The baseline length is about 600 km. The options have been set according to the recommendations in the help panels. The strategy `COMBINED` has been used.

At the beginning the program MAUPRP reports which measurements are marked. There are three reasons to mark the observation: low elevation of the satellite, unpaired observations, and small pieces of measurements (see Panel 4.4.2–2).

```
-----------------------------------------------------------------------
MARK UNPAIRED L1/L2 OBSERVATIONS: SUMMARY
-----------------------------------------------------------------------

SATELLITE    #L1 MARKED   #L2 MARKED
-----------------------------------------------------------------------

   19            60           0
   27            44           0
    2           134           0
   26           180           0
```

```
------------------------------------------------------------------------
MARK OBSERVATIONS WITH SMALL ELEVATION: SUMMARY
------------------------------------------------------------------------

SATELLITE    #L1 MARKED   #L2 MARKED
------------------------------------------------------------------------

   19             52           52
   27             72           72
    2             77           77
```

```
------------------------------------------------------------------------
MARK OBSERVATIONS WITHIN SMALL PIECES: SUMMARY
------------------------------------------------------------------------

SATELLITE    #L1 MARKED   #L2 MARKED
------------------------------------------------------------------------

   19              1           13
   27              2           15
    2              0           35
```

The first part of the program MAUPRP checks the double (exceptionally single) differences by smoothing (so-called non-parametric screening). Program produces the following output:

```
------------------------------------------------------------------------
CHECK DOUBLE DIFFERENCES: SUMMARY (WITH RESPECT TO REF.SATELLITE)
------------------------------------------------------------------------

SATELLITE    #OBS.   MARKED   SLIPS  INIT.SLIPS
------------------------------------------------------------------------

   19        600        0       0         3
   18        661        0       1         4
   24        812        3       3         4
   16        728        0       0         6
```

The second part of the program is the triple difference solution:

```
------------------------------------------------------------------------
TRIPLE DIFFERENCE SOLUTION
------------------------------------------------------------------------

FREQUENCY OF TRIPLE DIFF. SOLU.:           3
NUMBER OF TRIPLE DIFF OBS. USED:       13059
RMS OF TRIPLE DIFF SOLUTION (M):       0.008

COORDINATES  NEW-A PRIORI X (M):       0.116 +-       0.022
                          Y (M):      -0.010 +-       0.032
                          Z (M):       0.061 +-       0.016
```

In this example the a priori coordinates were very accurate. The difference between the a priori coordinates and the new values computed using the triple differences indicates the accuracy of the triple difference solution. The rms error of the triple difference solution should not be much larger than about 1 cm. Now, the triple difference residuals are screened. MAUPRP finds altogether 27 cycle-slips in this run:

```
------------------------------------------------------------------------
CYCLE SLIPS ACCEPTED IN THIS RUN
------------------------------------------------------------------------

NUMBER OF SLIPS IN L1:   27
NUMBER OF SLIPS IN L2:   27
```

```
NUMB  TYP N  EPOCH SAT FRQ WLF        SLIP   FRAC     RES.L3     IONOS
                                                       (M)        (M)
------------------------------------------------------------------------

   1  DUA *  2195   25  1   1     3820933.   0.00      0.000   -0.015
   2  DUA *             2   1     2977347.   0.00
                        5   1      843586.   0.02
   5  CLK *  2409  ALL  1   1    17724119.
   6  CLK *  2409  ALL  2   1    13810999.
```

The various types of cycle slip flags should be explained:

`DUA` means that the cycle slip was found by the dual band algorithm using the conditions (10.5) and (10.7),

`CLK` indicates so-called clock jumps (jumps on the single difference level, see above). Other possibilities (not in the example above) are

`SNG` which means that the cycle slip was found by the single frequency algorithm using the condition (10.7) only and

`USR` which indicates a cycle slip introduced by the user in interactive mode.

The areas which were changed in the most recent run are marked by an asterisk in the column "N". A long list of the pieces of measurements marked or changed follows:

```
------------------------------------------------------------------------
NEW OR MODIFIED MARKED AREAS IN THIS RUN
------------------------------------------------------------------------

NUMBER OF MARKED AREAS IN L1:  840
NUMBER OF MARKED AREAS IN L2:  710

NUMB   TYP N      EPOCHS      SAT   FRQ   #EPOCHS
------------------------------------------------------------------------

   1   UNP *      1 -     2    26    1        2
   2   UNP *      1 -     6    24    1        6

   3   ELV *      3 -     6    26    1        4
   4   ELV *      3 -     6    26    2        4

 178   GAR *    238 -   238    23    1        1
 179   GAR *    238 -   238    23    2        1



1459   DUA *    243 -   243     9    1        1
1460   DUA *    243 -   243     9    2        1
```

The possible marking types are:

- `DUA` dual band algorithm, see above,
- `SNG` single band algorithm, see above,
- `USR` user-defined or defined in `SATCRUX` file (see Chapter 8)
- `UNP` marked epochs with unpaired observations (L_1 without L_2 or vice versa),
- `ELV` observations at low elevation,
- `GAR` small pieces of observations (garbage), and
- `O-C` observations marked due to large observed–computed values during the triple difference solution (see Panel 4.4.2–2)

MAUPRP also gives the information on the ambiguities set up. Each satellite has one ambiguity corresponding to the first epoch. All other ambiguities are called multiple. Only the multiple ambiguities are listed. The ambiguities which were introduced in the most recent run are marked by an asterisk.

```
------------------------------------------------------------------------
MULTIPLE AMBIGUITIES
------------------------------------------------------------------------

NUMB  TYP  SATELLITE  EPOCH
------------------------------------------------------------------------

  1  *GAP      19     2395
  2  *GAP      19     2458
  3  *GAP      27     2395
```

There are the following types of multiple ambiguities:

- `FIL` ambiguity which was already set in the observation file header

CYC ambiguity which was introduced due to a cycle slip flag in the observation (see Panel 4.4.2–4, option IF CYCLE SLIP FLAG SET)

USR ambiguity which was introduced by the user (in interactive mode),

GAP ambiguity which was introduced due to a gap in the observations, and

PRP ambiguity which was introduced due to the detection of a cycle slip that could not be corrected (and outlier rejection was not possible).

At the end MAUPRP writes the very important message

```
FILE SAVED
```

which says that all the changes were written into the observation file. If "FILE NOT SAVED" is printed it means that *no* change were done to the original single difference file(s) (see Panel 4.4.2–1, option SAVED SCREENED FILES).

Example 2

The second example stems from the processing of the Turtmann campaign (see Section 4.2). The baseline was very short (only 2 km). We use the strategy BOTH in this example. All other options were identical to those in Example 1 with one exception: the maximal ionosphere difference in Panel 4.4.2–4 was set to 30 %. The strategy BOTH should not be used for baselines longer than about 10 km. However, this strategy may be superior to the COMBINED strategy if the baseline is very short and the receiver is of poor quality and provides measurements with a high noise level (of course, this was not the case of this example). The output from the program is similar to that of Example 1. The only difference is the type SNG (instead of DUA) assigned to detected cycle slips.

```
-----------------------------------------------------------------------
CYCLE SLIPS ACCEPTED IN THIS RUN
-----------------------------------------------------------------------

NUMBER OF SLIPS IN L1:    0
NUMBER OF SLIPS IN L2:    1

NUMB  TYP N  EPOCH SAT FRQ WLF        SLIP   FRAC     RES.L3     IONOS
                                                        (M)       (M)
-----------------------------------------------------------------------

   1  SNG *  1383   23  2   1     2397255.  -0.01

-----------------------------------------------------------------------
```

Satellite PRN 24 had a manoeuvre just in the time span of our example. The observations were marked using the SATCRUX file (see Section 10.7). MAUPRP reports these observations with marking type USR:

```
----------------------------------------------------------------------
NEW OR MODIFIED MARKED AREAS IN THIS RUN
----------------------------------------------------------------------

NUMBER OF MARKED AREAS IN L1:    9
NUMBER OF MARKED AREAS IN L2:    5

NUMB   TYP N      EPOCHS      SAT   FRQ   #EPOCHS
----------------------------------------------------------------------

   3   USR *      1 -  2878    24    1     2878
   4   USR *      1 -  2878    24    2     2878
```

10.6 Screening of Post-Fit Residuals

There are two programs in the Bernese GPS Software Version 4.0 which screen the residual files. The residual files may be generated by the programs GPSEST, MAUPRP, CODSPP, ORBGEN, IONEST, and RNXCYC. The residual files generated by the programs listed above are unformatted binary files containing all the residuals of one program run. There are two different types of residual files:

Type 1: Only linear combinations of L_1 and L_2 residuals are stored and may be displayed.

Type 2: L_1 and L_2 residuals are stored and may be displayed separately or in any linear combination.

The residual files may contain residuals on the zero difference level as well as on the single or double difference level.

The program REDISP (Menu 5.3.1) is an interactive program. REDISP first prompts the user for the units of the residual representation, then a table with all the file names originally processed is shown, from which the user may select the file he wants to display. After several user prompts (the dialog is self–explanatory) the residuals are displayed or stored in an output file (if an output file name was specified) in a readable format.

The program RESRMS (Menu 5.3.2) is a batch program. It screens the selected residual files and writes two output files. The first one is a summary file (extension ".SUM") which gives a nice overview per baseline and per satellite of the rms of the residuals. The second output file is the so-called edit file (extension ".EDT") which contains the list of points which have been identified as outliers. This edit file can be used with the program SATMRK (see below) to mark the outliers in the observation files.

10.7 Marking of Observations

All the observations used in the Bernese GPS Software Version 4.0 are stored in observation files (code or phase observations, zero or single differences). It is possible to set a so-called marking flag for each observation (the other flag used is the so-called cycle slip flag). If the marking flag is set no program will use the corresponding observation. It is also possible to reset the marking flags again.

The marking flags are used to mark outliers, observations at low elevation, small pieces of observations, etc.

There are several programs in the software which mark observations:

CODCHK marks the zero difference code measurements,

CODSPP marks both, phase and code observations where no receiver clock corrections could be estimated, CODSPP does *not* mark the outliers in the code observation files (although the outliers are not used in the program internally),

MAUPRP which marks the observations with low elevations, small pieces of observations and the observations suspected to be corrupted with cycle-slip (see Section 10.5), and

SATMRK (Menu 5.1 and the option “M”). The user may specify the satellite(s) and epochs which should be marked or it is possible to use an edit file e.g. stemming from the program RESRMS (see above).

It should be added, that the program SNGDIF does not use marked zero difference observations at all.

There is one more possibility how to prevent the programs (e.g. CODSPP, MAUPRP and GPSEST) from using some measurements. This possibility is the so-called “`SATCRUX`” file located in the general file directory:

```
SATELLITE PROBLEMS: MANOEUVRES OR BAD OBSERVATION INTERVALS          12-JUN-92
--------------------------------------------------------------------------------

SATELLITE  PROBLEM    ACTION           FROM                    TO
   **         *         *       YYYY MM DD HH MM SS  YYYY MM DD HH MM SS

   24         0         0       1993 09 27 11 45 00
    1         3         1       1993 10 03 23 00 00  1993 10 06 12 00 00
    5         3         1       1993 09 27 00 00 00  1993 10 01 24 00 00
   24         3         1       1993 09 27 00 00 00  1993 10 01 24 00 00
   31         0         0       1993 11 01 00 00 00
   29         0         0       1993 11 04 01 45 00
    7         0         0       1993 12 16 09 12 00
    1         0         0       1995 01 18 12 00 00
    9         3         1       1996  2 23 00 00 00  1996  2 23 24 00 00
    9         0         0       1996  2 23 12 00 00

PROBLEM:  MANOEUVRE=0, PHASE=1, CODE=2, CODE+PHASE=3
ACTION :  NEW ARC=0, MARK=1, REMOVE=2
```

The user may mark the measurements of specified satellites. The other important usage of the “`SATCRUX`” file is the setting of a new arc for the satellite (usually due to manoeuvre). This topic is discussed in Chapter 8.

11. Station Coordinates and Velocities

In GPS analysis station coordinates and, to a lesser extent, also station velocities play a dominant role because GPS is used by the majority of users to estimate (high accuracy) coordinates. Because GPS is an interferometric technique good a priori coordinates for at least one (reference) site have to be known. The user has the make sure that the orbit, the earth orientation parameters, and the station coordinates are given in one an the same reference frame. Chapter 18 deals with the combination of solutions of different sessions. Here we will only discuss aspects of session solutions.

11.1 Reference Frames

We use GPS as an interferometric technique which means that station coordinates are usually estimated "differentially", with the exception of global solutions. This implies that good coordinates for at least one station should be known in the correct reference frame in order to be able to obtain accurate coordinates for other sites in the same reference frame. Because reference frames have seven degrees of freedom (three translations, three rotations, and a scale factor) it is even preferable to have at least three stations with accurately known a priori coordinates. This however, does depend on the size of the network, the number of available sites, and their distance to the network. Normally, the a priori coordinates of the known sites are fixed or at least tightly constrained.

Most GPS users do no longer try to improve the orbits of the GPS satellites since the IGS has started to make available very precise satellite orbits. When no orbit improvement is performed the user has to make sure that the coordinates, the orbits, and the earth orientation parameters (EOPs) are given in the same reference frame. The EOPs are necessary to transform the (IGS) precise ephemerides from the Earth-Fixed reference frame to the inertial reference frame. The inertial reference frame is used for the numerical integration of the orbits. The consistency between coordinates, orbits, and EOPs is *essential*.

The broadcast ephemerides of the GPS satellites refer to the so-called WGS-84 reference frame, whereas the precise ephemerides of the IGS are given in an International Terrestrial Reference Frame (ITRF, since GPS week 0860 the ITRF94). The main difference between the two systems (WGS-84 and ITRF) is that the WGS-84 may only be realized by the users with a quality of about 1 meter geocentric position (because of the quality of the broadcast orbits and satellite clocks). The ITRF may be realized with centimeter accuracy if IGS orbits and ITRF coordinates of the IGS sites are included in the processing. The two systems are therefore consistent at about the 1 meter level. For both orbit types the user may take ITRF coordinates for his reference stations. When using IGS orbit products one has to check in which realization of the ITRF they are given (e.g. ITRF92, ITRF93, or ITRF94). This is indicated in the header of the precise orbit file. Furthermore one has to make sure to use the

EOP information which belongs to the particular orbits. For the IGS final orbits before GPS week 0860 one should use the IERS C04 EOP series. Since GPS week 0860 the IGS final orbits are created using a combined (IGS) pole which is made available together with the orbit. The individual IGS Analysis Centers, like CODE, are making available a weekly pole file together with the seven daily orbit files of the same week.

All necessary reference frame information like the ITRF94 coordinates, the IERS C04 and Bulletin A EOP series, and the CODE orbits with their respective EOP files may be found in the anonymous ftp account at the AIUB (see Chapter 7).

11.2 Coordinate Estimation

Normally, the a priori coordinates of the chosen reference site(s) are fixed or at least tightly constrained when processing a baseline or network with programs GPSEST or ADDNEQ. With the Bernese GPS Software version 4.0 we advise to constrain the sites rather than to fix them because the coordinates of the fixed sites will not be saved in the normal equation files. The normal equation files may be by program ADDNEQ where all constraints which were used in the original GPSEST solution, may be removed/altered and individual GPSEST solutions may be combined. See Chapter 18 for more information on the combination of solutions.

There are certain risks when fixing or constraining station positions because station coordinates or reference frame may be incorrect! An error analysis of the biases introduced into the solution when using incorrect station positions may be found in [*Beutler et al.*, 1988]. A bias of 1 m in height of a fixed site will cause a (small) scale effect of about 0.03 ppm. A bias in the horizontal components of the coordinates of the fixed site(s) will cause a rotation of the GPS network. To avoid such errors at least one site with well-established geocentric coordinates should be included in the local or regional network.

It is also possible to generate so called "fiducial free" network solutions. In the fiducial free network approach only loose constraints (1m–1km) are applied to the (reference) sites. The coordinates of the reference site do not have to be know exactly because they are only loosely constrained. Therefore practically all available stations may be selected as reference sites. The advantage of this procedure is that the solution will not be distorted due to biases in the a priori coordinates, the main disadvantage that the resulting coordinates, and other estimated parameters, are not in a well-defined reference frame. The results will also show considerable day to day variations because significant translations and rotations will exist between daily coordinate sets. Therefore the results of a fiducial free network solution have to be transformed into the appropriate reference frame using e.g. Helmert transformations. It is of course necessary for this transformation to use known station again. The (mandatory) daily transformations may also remove part of the geodynamical signal contained in the time series. The fiducial free strategy has mainly been used for global networks. Global networks have the advantage that the scale of the solution and the station heights are quite well defined. For local networks this is not the case. More information about constraining and combining solutions is given in Chapter 18.

With the Bernese GPS Software Version it is also possible to estimate the Earth's center of mass. This is only meaningful for global networks in combination with orbit estimation spanning a long observation time period.

The capability to generate pseudo-kinematic solutions has been added to the Bernese GPS Software. Because the partial derivatives for the station position are computed using the a priori station coordinates, the distance of a moving receiver should stay within the linear regime of the partial derivatives. This means that the moving receiver may not change its position by more than a few meters. Therefore the "kinematic solution" will not work properly for fast moving receivers like e.g. airplanes or cars. The pseudo-kinematic option is rather thought for applications like e.g. earthquake monitoring.

11.3 Site Displacements

The effects of the solid Earth tides have to be taken into account because they are at least one order of magnitude above the accuracies currently obtained for GPS-derived coordinates. In the Bernese GPS Software Version 4.0 we model the Solid Earth tides according to the IERS Standards 1992, [*McCarthy*, 1992]. However, only the so-called "step 1" corrections are implemented. The "step 2" corrections are currently neglected.

Other effects causing site displacements like ocean loading, polar tides, and atmospheric loading are also neglected at present. These neglected effects are rather small but the GPS-derived coordinate accuracies are approaching the level of these effects which is why they will be included in the next release of the Bernese GPS Software Version 4.0

The station coordinates are also moving in time due to plate motions. When using ITRF station coordinates one should also use the corresponding ITRF station velocities to map the station coordinates to the epoch of the GPS observations. For this purpose program COOVEL is available but it is not (yet) implemented in the menu system. For small networks plate motions do not play a very significant role because all stations move in the same direction. Exceptions may be found in plate boundary zones like e.g. the Mediterranean area.

If a site is not available in the list of ITRF coordinates and velocities we use the best estimate for the station velocity using the Nuvel-1 no-net-rotation plate motion model. Nuvel velocities may be computed using the program NUVELO. Both programs, COOVEL and NUVELO (both not yet included in the menu system) may be run using the `RUNGPS` command, see Chapter 3.

11.4 Coordinate Comparisons

The programs COMPAR and HELMR1 may be used to compare different coordinate solutions. The program COMPAR may also be used to combine coordinate sets using the full station covariance matrix. Because program ADDNEQ has similar, and more flexible capabilities, both programs are discussed in Chapter 18.

Program HELMR1, Menu 5.4.2, allows to compare two coordinate sets estimating a maximum of seven Helmert transformation parameters: three translations, three rotations, and a scale factor. This program is interactive, the user has the possibility to mark and/or exclude stations, and to change the number of transformation parameters to be estimated. A batch version of this program, which is called HELMER, is also available.

```
5.4.2                    SERVICES: HELMERT TRANSFORMATION

   CAMPAIGN          > DOCU40_1 <     (blank for selection list)

  Input Files:
    COORDINATES 1      > EQ_96165 <  Ref. Coo. (blank for selection list)
    VELOCITIES  1      > NO       <  Ref. Vel. (NO, blank for selection list)
    COORDINATES 2      > EQ_96166 <  Comp.Coo. (blank for selection list)

    USE STATION LIST   > NO       <     (NO: not used, blank: sel.list)

  Output File:
    HELMERT            > NO       <     (NO, if not to be created)
    TRANSFORMED COO. 2 > NO       <     (NO, if not to be created)
                                        (only for Coord. System GEOCENTRIC)
```

Apart from specifying two coordinate files for the comparison the user may also specify a velocity file to map the first coordinate set to the epoch of the second coordinate set. This, of course, is only useful if some or all sites show significant displacements in the time interval between the two coordinate sets. Furthermore, a file containing a list of stations may be specified. The stations specified in this list will be used for the Helmert transformation. In the interactive mode the exclusion and marking of stations may still be altered. Finally, two output files may be specified. The first contains an overview of the Helmert transformation including residuals for all stations and the transformation parameters (see Chapter 4 for an example of the Helmert output); the other contains the coordinates of the second coordinate set after transformation, so it contains the second set of coordinates given in the reference frame of the first set.

11.5 Merging Coordinate Files

The program CRDMRG, Menu 5.4.5 may be used to merge different coordinate files into one master file. The program was developed mainly for use within the BPE environment in order to update coordinates of new sites. With the Bernese GPS Software coordinates may be obtained in several ways, which is indicated in the coordinate file with a flag (see Chapter 23 for a description of the coordinate flags).

The coordinates may be taken from the RINEX file (RXOBV3), from code single point positioning (CODSPP), from the data cleaning step (MAUPRP), or from the parameter estimation programs GPSEST or ADDNEQ. With the program CRDMRG you may add the coordinates of a (new) station easily to a “master” coordinate file by merging a coordinate file obtained e.g. by CODSPP with the existing “master” file. Based on the coordinate flags the coordinates in the master file will be updated or not. If e.g. the master file contains a GPSEST estimate for a particular station its coordinates will <u>not</u> be updated it if you are merging coordinate results from a CODSPP run. However, if you are merging coordinate results from an ADDNEQ run, the coordinates of this particular station will be updated.

12. Troposphere Modeling and Estimation

12.1 Motivation

In view of the fact that orbit errors must no longer be considered as an important error source (thanks to the availability of high accuracy orbits through the International GPS Service for Geodynamics (IGS), see Chapter 8), propagation delays of the GPS code and phase signals due to the *neutral atmosphere, i.e. the troposphere*, probably are the *ultimate accuracy limiting factor* for geodetic applications of the GPS.

Let us distinguish two kinds of troposphere biases:

- Relative troposphere biases caused by errors (unmodeled effects) of tropospheric refraction at one of the endpoints of a baseline relative to the other endpoint.
- Absolute troposphere biases caused by errors (unmodeled effects) of tropospheric refraction *common to both endpoints* of a baseline.

Both error sources are dealt with in detail in [*Beutler et al.*, 1988]. It is remarkable that *relative troposphere biases* invoke primarily *biased station heights* whereas *absolute troposphere biases* produce *scale biases* of the estimated baseline lengths.

For local and smaller regional campaigns relative troposphere errors are much more important and more difficult to model. To a first order the station height bias may be computed as

$$\Delta h = \frac{\Delta \varrho_r^0}{\cos z_{max}} \tag{12.1}$$

where

Δh ... is the induced station height bias,
$\Delta \varrho_r^0$ ... is the relative tropospheric zenith correction, and
z_{max} ... is the maximum zenith angle of the observation scenario.

In the above order of magnitude formula it is assumed that the satellites are uniformly distributed over the sky above the observing sites. Due to the fact that the GPS orbits all have inclinations of

55° with respect to the Earth's equator this assumption is not true, actually. [*Santerre*, 1991] studies this effect in particular.

In any case eqn. (12.1) indicates, that a bias of only 1 cm in the relative troposphere leads to an error of approximately 3 cm in the estimated relative station height!

According to [*Beutler et al.*, 1988] the corresponding formula for the impact of an absolute troposphere error reads as

$$\frac{\Delta\ell}{\ell} = \frac{\Delta\varrho_a^0}{R_e \ \cos z_{max}} \tag{12.2}$$

where

ℓ, $\Delta\ell$ ... are the baseline length and the associated bias,
$\Delta\varrho_a^0$ ... is the absolute troposphere bias in zenith direction (common to both endpoints of the baseline), and
R_e ... is the Earth's radius.

Eqn. (12.2) says that an absolute troposphere bias of 10 cm induces a *scale bias* of 0.05 ppm, a relatively small effect compared to the height error caused by a relative troposphere bias. Nevertheless the effect should be taken into account for baselines longer than about 20 km. Again, a uniform satellite distribution in a spherical shell centered above the stations down to a maximum zenith distance of z_{max} was assumed when deriving formula (12.2). The consequences of a non-uniform distribution were studied by [*Santerre*, 1991].

In a certain sense an absolute troposphere error is very similar to an error caused by the ionosphere. The main difference between the two effects is due to the circumstance that tropospheric refraction is produced in the lowest levels of the atmosphere (99 % below 10 km) whereas the *ionospheric shell height* is about 400 km. Tropospheric refraction tends to be much more site specific than ionospheric refraction for that reason.

In summary, we may state that troposphere biases are orders of magnitude above the noise level of the phase observable. Their influence thus must be reduced to make full use of the accuracy of the observable by either of the following two methods:

- Model tropospheric refraction *without* using the GPS observable (e.g. by using ground met measurements or water vapour radiometers).
- Model the tropospheric zenith delay in the general GPS parameter estimation process.

Both methods are widely used today, for both methods there are options in the Bernese GPS Software Version 4. Before discussing the options available we briefly review some aspects of the theory.

12.2 Theory

Tropospheric refraction is the path delay caused by the neutral (non-ionized) part of the Earth's atmosphere. The troposphere is a *non-dispersive medium* for radio waves up to frequencies of about 15 GHz (see e.g. [*Baueršíma*, 1983]). Tropospheric refraction is thus identical for both GPS carriers,

L_1 and L_2 (and both, phase and code, measurements – see eqn. (9.14)). The tropospheric path delay is defined by

$$\Delta\varrho = \int (n-1)\,\mathrm{d}\,s = 10^{-6} \int N^{trop}\,\mathrm{d}\,s\ , \tag{12.3}$$

where n is the refractive index and N^{trop} the so-called refractivity. The integration has to be performed along the actual path of the signal through the atmosphere. According to [*Hopfield*, 1969] it is possible to separate N^{trop} into a *dry* and a *wet* component

$$N^{trop} = N_d^{trop} + N_w^{trop}\ , \tag{12.4}$$

where the dry component is due to the dry atmosphere and the wet component due to the water vapor in the atmosphere. About 90 % of the path delay due to tropospheric refraction stems from the dry component [*Janes et al.*, 1989]. Using the last equation we may write

$$\Delta\varrho = \Delta\varrho_d + \Delta\varrho_w = 10^{-6} \int N_d^{trop}\,\mathrm{d}\,s + 10^{-6} \int N_w^{trop}\,\mathrm{d}\,s\ . \tag{12.5}$$

According to [*Essen and Froome*, 1951] we have

$$N_{d,0}^{trop} = 77.64\,\frac{p}{T}\left[\frac{\mathrm{K}}{\mathrm{mb}}\right] \quad \text{and} \quad N_{w,0}^{trop} = -12.96\,\frac{e}{T}\left[\frac{\mathrm{K}}{\mathrm{mb}}\right] + 3.718\cdot 10^5\frac{e}{T^2}\left[\frac{\mathrm{K}^2}{\mathrm{mb}}\right]\ , \tag{12.6}$$

where p is the atmospheric pressure in millibars, T the temperature in degrees Kelvin and e is the partial pressure of water vapor in millibars. The coefficients were determined empirically.

The tropospheric delay depends on the distance traveled by the radio wave through the neutral atmosphere and is therefore also a function of the satellite's zenith distance z. To emphasize this elevation-dependence the tropospheric delay is written as the product of the delay in zenith direction $\Delta\varrho^0$ and the so-called *mapping function* $f(z)$:

$$\Delta\varrho = f(z)\ \Delta\varrho^0\ . \tag{12.7}$$

According to e.g. [*Rothacher*, 1992] it is better to use different mapping functions for the dry and wet part of the tropospheric delay:

$$\Delta\varrho = f_d(z)\ \Delta\varrho_d^0 + f_w(z)\ \Delta\varrho_w^0\ . \tag{12.8}$$

Below, we will give a list of the *a priori models for tropospheric refraction* available in the Bernese GPS Software Version 4. Each model has its own mapping function(s). It is worth mentioning, however, that to a first order (*flat earth society*) all mapping functions may be written as:

$$f_d(z) = f_w(z) = f(z) = \frac{1}{\cos z}\ . \tag{12.9}$$

The following models to take into account tropospheric refraction are available in the Bernese GPS Software Version 4:

- the **Saastamoinen model** [*Saastamoinen*, 1973],
- the **modified Hopfield model** [*Goad and Goodman*, 1974],
- the **simplified Hopfield model** [*Wells*, 1974], and

- the **differential refraction model based on formulae by Essen and Froome** [*Rothacher et al.*, 1986].

More models will be available in the near future.

Usually, we take the Saastamoinen model as *a priori model* to account for tropospheric refraction. This model is based on the laws associated with an ideal gas. [*Saastamoinen*, 1973] gives the equation

$$\Delta\varrho = \frac{0.002277}{\cos z} \left[p + \left(\frac{1255}{T} + 0.05 \right) e - \tan^2 z \right] , \tag{12.10}$$

where the atmospheric pressure p and the partial water vapor pressure e are given in millibars, the temperature T in degrees Kelvin; the result is in given meters. [*Baueršíma*, 1983] gives special correction terms B and δR:

$$\Delta\varrho = \frac{0.002277}{\cos z} \left[p + \left(\frac{1255}{T} + 0.05 \right) e - B \tan^2 z \right] + \delta R . \tag{12.11}$$

The correction term B is a function of the height of the observing site, the second term δR depends on the height and on the elevation of the satellite. Only the former term is implemented in the present version of our software.

In the model either *measured values for pressure, temperature, and humidity* or the *values derived from a standard atmosphere model* may be used. If you decide to use surface met values stemming from a model atmosphere, the following height-dependent values for pressure, temperature and humidity are assumed [*Berg*, 1948]:

$$\begin{aligned} p &= p_r \cdot (1 - 0.000226 \cdot (h - h_r))^{5.225} \\ T &= T_r - 0.0065 \cdot (h - h_r) \\ H &= H_r \cdot e^{-0.0006396 \cdot (h - h_r)} \end{aligned} \tag{12.12}$$

where p, T, H are pressure, temperature (Kelvin), and humidity at height h of the site; p_r, T_r, H_r are the corresponding values at reference height h_r. The reference height h_r, and the reference values p_r, T_r, H_r are defined in the file `X:[GEN]CONST.` and we do *not* recommend to change these values:

$$\begin{aligned} h_r &= 0 \text{ m} \\ p_r &= 1013.25 \text{ mbar} \\ T_r &= 18^o \text{ Celsius} \\ H_r &= 50\ \% \end{aligned} \tag{12.13}$$

12.3 Using Ground Meteorological Data

Let us first discuss the implications of small biases in ground met data (pressure, temperature, humidity) on the estimated station heights.

Table 12.1, together with formula (12.1), give an impression of the sensitivity of the estimated station height (independent of the baseline length!) on biases in surface met measurements for different atmospheric conditions. We see e.g. that in a hot and humid environment (last line in Table 12.1) an error of only 1% in the relative humidity will induce a bias of 4 mm in the tropospheric zenith delay, which will in turn produce (using equation (12.1)) a height bias of *more than one centimeter*! It is common knowledge that it is virtually impossible to measure the relative humidity to that accuracy; moreover the measured humidity is usually *not* representative for the entire environment of a site. This is why **experience tells that the estimation of troposphere parameters is a necessity if highest accuracy is required and if only ground met data are available**. Similar remarks are true for temperature measurements. It should be possible, on the other hand, to measure surface pressure to the accuracy level necessary (0.1 mm) to keep pressure-induced height errors harmless.

T C^o	P mbar	H %	$\left\|\frac{\partial\Delta\varrho}{\partial T}\right\|$ mm/C^o	$\left\|\frac{\partial\Delta\varrho}{\partial P}\right\|$ mm/mbar	$\left\|\frac{\partial\Delta\varrho}{\partial H}\right\|$ mm/1%
0^o	1000	100	5	2	0.6
30^o	1000	100	27	2	4
0^o	1000	50	3	2	0.6
30^o	1000	50	14	2	4

Table 12.1: Tropospheric Zenith Delay as a Function of Temperature T, Pressure P, and relative Humidity H.

You should always keep in mind the orders of magnitude reflected in Table 12.1 when using ground met data. Our conclusion is, that *only if you are able to provide met values stemming from Water Vapour Radiometers you have a good chance to get around the estimation of tropospheric zenith delays*. There is one exception to that rule: If you are working in a small network (diameter $<$ 10 km) in a flat Earth environment with height differences $<$ 100 m (e.g. in the Netherlands) you may be best advised by *not* using surface met information (using the a priori atmosphere model defined in the software) and by *not* estimating troposphere parameters.

12.4 Introducing Troposphere Data into the Processing

Three programs in the Bernese GPS Software model tropospheric refraction:

CODSPP (see Chapter 10) may model tropospheric refraction using either the Saastamoinen or the Hopfield model. The values for pressure, temperature, and humidity are taken from the standard atmosphere (see Section 12.2) using the reference values given in the file `X:[GEN]CONST.` It is not possible to introduce ground met data. If only poor a priori coordinates are available it may be wise *not* to apply tropospheric refraction model.

MAUPRP (see Chapter 10) uses the Saastamoinen model with the standard atmosphere values. It is not possible (and not necessary) to select the model in this program.

GPSEST, the main parameter estimation program, has many options to deal with the tropospheric refraction. The user has to decide:

1) in Panel 4.5–2 which a priori model should be used for tropospheric refraction (there are four models available at present – see Section 12.2),

2) which values for temperature, pressure, and humidity should be used (the values may stem either from the standard atmosphere or from ground met measurements),

3) whether corrections with respect to the selected a priori model should be estimated.

We discussed the a priori models and the standard atmosphere in Section 12.2. In this section we give an overview of the met data file types which may be introduced into the processing. The estimation of tropospheric parameters will be discussed in the last section of this chapter.

When preparing a GPSEST run the user may specify the met files in Panel 4.5. It is possible to specify a list of met files. Each file has to contain the data for exactly one station covering the time span of the entire session(s). However, it is *not* necessary to specify the met file for each station (for stations without met data the a priori troposphere correction will be computed using the standard atmosphere). The met files have the default extension `.MET` and they are located in campaign-specific `ATM` directory. You may either prepare these files manually (using an ASCII editor), they may be transformed from RINEX met files using program RXMBV3 (see Chapter 7), or they may stem from a PRPMET run (see below). There are four types of met files (see also Chapter 23). The first type contains pressure, temperature, and humidity values:

```
DISTRIBU           BERNESE MET.FILES
STATION : ZIMMERWALD GPS87   UTC-LOCAL TIME(HOURS) =  0 TYP= 1
JJJJ MM DD HH MM SS  PPP.PP  TT.TT  HH.HH
1989 10 14 18  0  6  911.40  11.20  72.10
1989 10 14 18 30  5  911.90  10.40  69.30
...
1989 10 15  6  0  5  915.60   7.10  84.60
```

The second type contains dry and wet temperature:

```
EXAMPLE FOR DRY AND WET TEMPERATURE (NOT REALISTIC !)
STATION : ZIMMERWALD         UTC-LOCAL TIME(HOURS) =  0 TYP= 2
JJJJ MM DD HH MM SS  PPP.PP  TDRY   TWET
1987  6 16 10 30  0  910.49  12.61  17.41
1987  6 16 10 32  0  907.60  12.94  22.29
...
1987  6 16 10 40  0  903.21  14.11  21.36
```

The third type contains directly the total tropospheric zenith delays:

```
EXAMPLE OF A ZENITH DELAY FILE
STATION : ZIMMERWALD         UTC-LOCAL TIME(HOURS) = -1 TYP= 3
JJJJ MM DD HH MM SS  ZENITH DELAY (M)
1987  6 16 10 00  0     2.100
1987  6 16 10 30  0     2.115
...
```

Be aware, that if a met file of type 3 is specified, no a priori troposphere model is used and the tropospheric delay in eqns. 9.14 will be simply the value given in the file (interpolated for current epoch) divided by $\cos z$. The last type of met file contains *the zenith delay corrections* with respect to an a priori model:

```
ADDNEQ: 3-DAY 230, AMB. FIXED, POLE: 2 PAR/3 DAYS ABS            22-AUG-96 06:10
STATION : ZIMM 14001M004     UTC-LOCAL TIME(HOURS) =  0 TYP= 4  #VALUES=  1  MOD= -1
 JJJJ MM DD HH MM SS  DDDD.DDDD
 1996  8 16 23 59 56     0.0687
 1996  8 17  6  0  0     0.0687
 ...
 1996  8 18  0  0  3     0.0927
```

The corrections with respect to the a priori model may be estimated in GPSEST or in ADDNEQ – see next section. These estimations may be stored in tropospheric files (Panel 4.5–0) with the default extension .TRP. One file (containing all stations) is generated by one GPSEST run. If you want to introduce these estimations into a subsequent run, it is necessary to generate *.MET files (one per station) from the .TRP file. This is possible with the program PRPMET. However, you cannot start this program using the menu system. It is necessary to prepare the N– and F–file manually (using an ASCII editor). Examples for the input files (PRPMETN.INP and PRPMETF.INP) may be found in X:[INX].

CODE daily estimates of the troposphere are available through anonymous ftp (ubeclu.unibe.ch) in the directory AIUB$FTP:[BSWUSER.ATM] for all the global sites processed by CODE. If you introduce stations from the IGS network into your processing (the reason might be e.g. the correct reference frame for you local network) it is a good idea to introduce the tropospheric delay estimates for these stations into your solutions, too. When making consistent use of the CODE coordinates, orbits, Earth's orientation parameters, and troposphere estimates for the IGS stations included into your network, you are able to get results that are almost identical to those you would obtain by processing your network data together with the global data.

12.5 Tropospheric Delay Estimation

We pointed out in Section 12.3 that, usually, the a priori model of the tropospheric delay is not sufficient if highest accuracy is required. Therefore it is necessary to *estimate* the troposphere in GPSEST (and ADDNEQ). We recommend to use an a priori model and to estimate only the *corrections* with respect to this a priori model.

Local Troposphere Models

You may estimate the parameters of a *local troposphere model*. This model assumes, that the correction with respect to the selected a priori model for a station at height h and a satellite at zenith angle z is given by

$$\frac{1}{\cos z} \sum_{i=0}^{n} a_i \, (h - h_r)^i \; , \qquad (12.14)$$

where the reference height h_r is taken from the constants file (X:[GEN]CONST.]), and a_i are the estimated parameters. Local troposphere models are *not* supported by the menu system. It is necessary to edit the file GPSESTI.INP before starting the program GPSEST (see Chapter 3). The relevant part of the input file looks like

```
LOCAL TROPOSPHERE MODEL PARAMETERS:
----------------------------------

      MODEL  #PAR.       FROM             TO
                        (YEAR,MONTH,DAY,HOUR)
F      ***    **    **** ** ** **.*   **** ** ** **.*

--> :    1     3    1987 10 28 00.0   1987 10 28 23.9
--> :    2     3    1987 10 29 00.0   1987 10 29 23.9
--> :    3     3    1987 10 30 00.0   1987 10 30 23.9

      TERM   A PRIORI SIGMA

F      **     ******.*****

--> :   1          0.00001
        2      10000.00000
        3      10000.00000
```

In this example three local troposphere models (with polynomial degree $n = 2$) are estimated for three 24-hour sessions. The zero-degree term has to be constrained to zero for a local network (i.e. the estimation of an absolute tropospheric correction is not possible). We recommend to use local troposphere models only in *local* (distances between stations several kilometers) campaigns with big height differences.

Troposphere Parameters for Individual Stations

The estimation of troposphere parameters for *individual stations* is much more common than the estimation of local troposphere models. The total tropospheric delay correction $\Delta\varrho_k^i$ in eqns. (9.14) is given by

$$\Delta\varrho_k^i = \Delta\varrho_{apr,k}\ f_{apr}(z_k^i) + \Delta\varrho_k(t)\ f(z_k^i)\ , \qquad (12.15)$$

where

$\Delta\varrho_{apr,k}$	...	is the tropospheric zenith delay according to the a priori model specified. If standard atmosphere is used (no met files), this delay is time-invariant (depends on the station height only),
z_k^i	...	is the zenith distance (satellite i, station k),
f_{apr}	...	is the mapping function (each a priori model has its mapping function),
$\Delta\varrho_k(t)$	...	is the (time-dependent) troposphere parameter for station k, and
$f(z_k^i)$	...	is the mapping function used for the parameter estimation. This mapping function may be different from f_{apr}. The user has to select this mapping function in Panel 4.5–2.4.0.

Let us give you several recommendations concerning the estimation of troposphere parameters for individual stations:

- In *regional* or *global* campaigns it is recommended to estimate troposphere parameters for *all* stations.
- In *local* campaigns it is recommended to estimate troposphere parameters for *all but one* stations (due to strong correlations between troposphere parameters and stations heights in local campaigns).

- If tropospheric delays from global solutions are available (e.g. from CODE – see previous section) for some stations it is recommended to introduce these values and to estimate troposphere parameters only for the remaining stations of the regional or local campaign.
- If water vapour radiometry measurements and high precision barometers and thermometers are available you may generate a met file of type 3 by adding the dry and the wet components. Then you probably need not to estimate troposphere parameters.

As stated above, the troposphere parameters $\Delta\varrho_k(t)$ are time-dependent. In the Bernese GPS Software a set of parameters $\Delta\varrho_k$ may be estimated for each site, each parameter being valid within a time interval $\langle t_i, t_{i+1}\rangle$. Manually (by editing the `GPSESTI.INP` file) you may select the intervals almost arbitrarily. The menu system divides the entire session into intervals of equal length. The user has to specify the corresponding options in panel

```
4.5-2.4.0     PARAMETER ESTIMATION: SITE-SPECIFIC TROPOSPHERE PARAMETERS

   General Apriori Sigma:             Special Station Sigma: (0.0: NO EST.)
     ABSOLUTE    > 0.10   <  m          ABSOLUTE     > 0.0000 <  m
     RELATIVE    > 5.00   <  m          RELATIVE     > 0.0000 <  m

   Special Station Selection:
     STATIONS    > NONE            <      (blank for selection list, NONE,
                                           SPECIAL_FILE.. $FIRST, $LAST)
   Set-up of Parameters:
     INTERPRET NEXT VALUE AS    > NUM <       (NUM: num/sess; MIN: minutes)
     # PAR/SESS OR PAR INTERVAL > 4   <       (num/sess or minutes)

     MAPPING FUNCTION           > COSZ     <  (COSZ or HOPFIELD)
```

We refer to the help panel for details on all options. Please note that it is possible to constrain not only individual parameters to the a priori model value(= absolute constraints), but also the *difference* between two subsequent parameters of the same station to zero (= relative constraints). For details we refer to [*Rothacher*, 1992]. Estimating a large number of parameters $\Delta\varrho_k$, with tight relative constraints produces results similar to Kalman filter techniques.

13. Ionosphere Modeling and Estimation

13.1 Subdivision of the Atmosphere

The *atmosphere* is usually subdivided into two main shells, the *troposphere* and the *ionosphere*, because the signal propagation conditions are quite different in these two shells.

- The *troposphere* — also called the *neutral* atmosphere — is the lower part of the Earth's atmosphere which extends from the Earth's surface to an altitude of about 40 km. The signal propagation depends mainly on the temperature, the pressure, and the water vapor content of the atmospheric layers (see Chapter 12).
- The *ionosphere* is the upper part of the Earth's atmosphere, located approximately between 70 and 1000 km above the Earth. The signal propagation is mainly affected by free charged particles.

13.2 Motivation

You have to deal with the *ionospheric* refraction, i. e. with the term I of the observation equation (9.14), in the following processing steps:

(1) *single-point positioning* (program CODSPP), if you do *not* use the *ionosphere-free* linear combination (L3),

(2) *pre-processing* (program MAUPRP),

(3) *ambiguity resolution* (program GPSEST), if you do *not* make use of *precise* dual-band code measurements by analyzing of the Melbourne-Wübbena linear combination (9.28), and

(4) *parameter estimation* (program GPSEST), if you do *not* use the *ionosphere-free* linear combination (L3).

Note that ambiguity resolution is a special case of parameter estimation.

If you process a small network analyzing single-frequency data and disregard ionospheric refraction, you get a contraction of the network. The scale bias introduced in a GPS network by *unmodeled* ionospheric refraction is given in Table 13.1 (according to [*Beutler et al.*, 1988]) as a function of the linear combination (LC) and the maximum zenith distance $z_{\max}$ processed. This scale bias is proportional to the Total Electron Content (TEC), the total number of electrons in a rotation cylinder centered around the line of sight receiver–satellite with a cross section of 1 m^2. The TEC is expressed in so-called TEC Units (TECU). Example: For L1 solutions with an elevation cut-off angle of 15° and a TEC of 10 TECU, we may expect a baseline shrinking of about $10 \cdot 0.10 = 1.0$ ppm (or 1.0 mm/km).

LC	Scale factor in ppm/TECU			
	$z_{\max} = 80^{\circ}$	$z_{\max} = 75^{\circ}$	$z_{\max} = 70^{\circ}$	$z_{\max} = 65^{\circ}$
L1	-0.15	-0.10	-0.08	-0.06
L2	-0.24	-0.16	-0.12	-0.10
L5	$+0.19$	$+0.13$	$+0.10$	$+0.08$

Table 13.1: Ionosphere-induced scale factor (per TECU) when neglecting the ionosphere

When only single-frequency data is available, GPS-derived ionosphere models are very efficient in removing or greatly reducing the ionosphere-induced scale bias under homogeneous and moderate ionospheric conditions (see e. g. [*Wild*, 1994]). For very small high-precision networks (with a maximum extent of about 10 km), we recommend to use — even if dual-band data is available — L1 data only together with an ionosphere model (see e. g. [*Beutler et al.*, 1995]). Such ionosphere models are useful for other applications, too.

13.3 Theory

13.3.1 Introduction

The ionosphere may be characterized as that part of the upper atmosphere where a sufficient number of electrons and ions are present to affect the propagation of radio waves. The spatial distribution of electrons and ions is mainly determined by

- photo-chemical processes and
- transportation processes.

Both processes create different layers of ionized gas in different altitudes. The diagram indicating the number of ions produced as a function of altitude is called *Chapman profile*. This profile, which is a function of the solar zenith angle, is illustrated in Figure 13.1. Because of several transportation processes in the ionosphere, the actual electron concentrations may differ considerably from those estimated from the production layers.

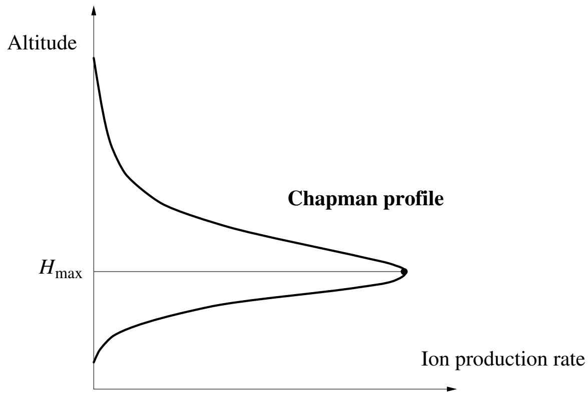


Figure 13.1: Chapman curve of ionization rate

The degree of ionization shows large variations which are correlated with the solar activity; geomagnetic influences play an important role, too. The solar activity may be characterized e. g. by the sunspot number, where one observes an 11-year cycle and in addition an 80- to 100-year super-cycle. Figure 13.2 shows the monthly smoothed sunspot numbers[1] from 1950 to 1995. We see that the most recent ionospheric maximum must have happened in 1989/1990 and that currently we are approaching a minimum. The situation will deteriorate because the Sunspot cycle will soon be in the increasing phase, with a maximum expected in 2001/2002.

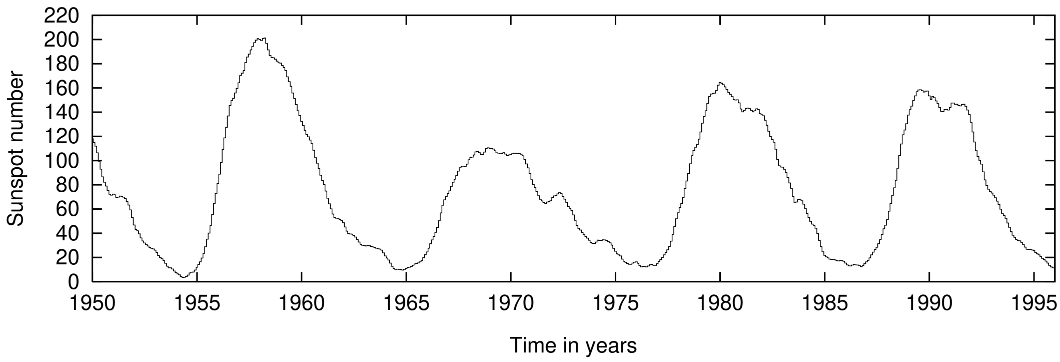


Figure 13.2: Monthly smoothed sunspot numbers

13.3.2 Characterizing of the Ionosphere

The state of the ionosphere may be described by the electron density n_e in units of electrons per m^3. The impact of the state of the ionosphere on the propagation of radio waves is characterized by the Total Electron Content E

$$E = \int_R^S n_e(s)\, ds. \tag{13.1}$$

[1] Data obtained from `http://www.oma.be/KSB-ORB/SIDC/` and `ftp://ftpserver.oma.be/pub/astro/sidcdata/`, respectively.

The integral contains the total number of free electrons that are included in a rotation cylinder with a cross-section area of 1 m^2, aligned along the signal path s between receiver R and satellite S. In geodetic applications the TEC E is measured in so-called TEC Units (TECU), where 1 TECU corresponds to 10^{16} electrons per square meter ($10^{16}/\mathrm{m}^2$). For comparisons, the vertical TEC E_v is formed as

$$E_\mathrm{v} = E \,\cos z', \tag{13.2}$$

where z' is the zenith distance of the signal path with respect to the vertical in a mean altitude of the ionospheric shell.

The *ionosphere* is a *dispersive* medium in the radio-band — as opposed to the *troposphere* (see Chapter 12). This implies that ionospheric refraction depends on the *frequency* of the signal considered. Neglecting higher-order terms we may write the ionospheric refraction coefficient for carrier phase measurements as

$$n_I = 1 - \frac{\alpha\, n_\mathrm{e}}{f^2}, \tag{13.3}$$

where α is a constant, n_e is the electron content along the signal propagation path, and f is the carrier frequency. Integration of (13.3) over the entire propagation path s taking into account eqn. (13.1) yields the total effect of ionospheric refraction on phase measurements

$$\Delta\varrho_I = \int_s (n_I - 1)\, ds = -\frac{\alpha\, E}{f^2} \qquad \text{with} \quad \alpha = 4.03\cdot 10^{17}\ \mathrm{m\,s^{-2}\,TECU^{-1}}, \tag{13.4}$$

where E is the slant TEC.

Formulae (13.3) and (13.4) indicate that the index of refraction, and thus the refraction effect, are proportional to the inverse of the squared frequency. Consequently, if two frequencies are available, the ionospheric delay may be eliminated by forming the so-called *ionosphere-free* linear combination (L3) according to eqns. (9.19) and (9.20).

In observation equation (9.14) we defined the term I which corresponds to the ionospheric delay on L1:

$$I = \frac{\alpha\, E}{f_1^2} \qquad \text{with} \quad f_1 = 1.57542\cdot 10^9\ \mathrm{s}^{-1}. \tag{13.5}$$

Hence the ionospheric delay may be written as

$$\Delta\varrho_I = \mp\frac{f_1^2}{f^2}\, I, \tag{13.6}$$

where we have to use the *negative* sign for *phase* observations and the *positive* sign for *code* observations. The resulting one-way range error $\Delta\varrho_I$, for GPS frequencies, may vary from less than 1 m to more than 100 m.

The neglected higher-order terms include the actual higher-order terms of the Taylor series, the ray-path-bending effect, and the effect of the geomagnetic field. According to [*Brunner and Gu*, 1991] these terms may reach — on zero-difference level — a few centimeters for low elevation angles and a very high electron content. Nevertheless, the ionosphere-free LC eliminating the first-order term is an excellent approximation especially on the double-difference level, where the residual range error (RRE), the difference between the ionosphere-free LC and the true range, is smaller than a few millimeters even for long baselines.

13.3.3 Influence of the Ionosphere on Different Linear Combinations

Table 13.2 gives an overview of the influences of two classes of errors on different linear combinations (LCs): the original carriers (L1 and L2), the ionosphere-free LC (L3), the geometry-free LC (L4), and the wide-lane (L5). We may distinguish between *systematic* errors like *geometrical* errors caused e. g. by the limited accuracy of troposphere and orbit representation ("geometry"), *ionospheric* errors ("ionosphere"), as well as *random* errors like measurement noise ("noise") or also multi-path effects.

LC	λ	κ_1	κ_2	Geometry		Ionosphere [TECU]		Noise	
						6.05	1.15		
	[m]	[m/m]	[m/m]	[m]	[cycles]	[m]	[cycles]	[m]	[cycles]
L1	0.190	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00
L2	0.244	0.00	1.00	1.00	0.78	1.65	1.28	1.00	0.78
L2′	0.122	0.00	1.00	1.00	1.56	1.65	2.57	1.00	1.56
L3	—	2.55	−1.55	1.00	—	0.00	—	2.98	—
L3 with N_5	0.107	2.55	−1.55	1.00	1.78	0.00	0.00	2.98	5.30
L4	—	1.00	−1.00	0.00	—	−0.65	—	1.41	—
L4 with N_5	0.054	1.00	−1.00	0.00	0.00	−0.65	−2.28	1.41	4.99
L5	0.862	4.53	−3.53	1.00	0.22	−1.28	−0.28	5.74	1.27
L5′	0.431	4.53	−3.53	1.00	0.44	−1.28	−0.57	5.74	2.54

Table 13.2: Influences of the most important error sources on different linear combinations

LCs marked with a dash (e. g. L2′) are formed when data from *squaring-type* receivers is processed, where L2 is available with *half* the wavelength λ_2 only. In this case the *wide-lane* (L5) ambiguities N_5 are formed according to

$$N_5' = 2\,N_1 - N_2' \qquad \text{with} \quad \lambda_5' = \lambda_5/2 = 0.431 \text{ m}. \tag{13.7}$$

Note that the above linear combination is superior to e. g. $N_5'' = N_1 - N_2'$ with $\lambda_5'' = 0.341$ m regarding the ionospheric influence. "L3 with N_5" denotes the so-called *narrow-lane* LC where we introduce the previously resolved ambiguities N_5 (or N_5'). κ_1 and κ_2 are the factors to form the particular LCs based on L1 and L2. All errors are given in *meters* and *cycles*, scaled to the error on the first carrier L1. Note that the information concerning the "noise" is based on two assumptions, namely:

- the measuring noise of L1 and L2 expressed in *meters* is of the same order and
- L1 and L2 are *not* correlated.

In this chapter errors related to the "ionosphere" are of major interest. We may recognize in Table 13.2 comparing the ionospheric errors expressed in *cycles* that the wide-lane linear combination (L5) is much less ionosphere-sensitive for ambiguity resolution than L1 and L2 (see also Chapter 15). The relation between an ionospheric error on a particular LC and slant TEC (in TECU) is also given in this Table. Example: An ionospheric bias of 1 cycle in L5 corresponds to $1.15/0.28 = 4.14$ TECU.

13.3.4 Ionospheric Effects on GPS Signals

On one hand irregularities in the ionosphere produce short-term signal variations. These scintillation effects may cause a large number of cycle slips because the receiver cannot follow the short-term signal variations and fading periods. Scintillation effects mainly occur in a belt along the earth's geomagnetic equator and in the polar auroral zone.

On the other hand a high electron content produces strong horizontal gradients and corrupts the ambiguity solution with geometrical methods. The only reliable strategy to solve the ambiguities in these cases is the Melbourne-Wübbena linear combination (9.28) using data from P-code receivers. This method very much depends on the quality of P-code measurements, which are often poor under Anti-Spoofing (AS) conditions. A very high electron content only occurs in equatorial regions.

As a result of this, we classify ionospheric refraction into

- a *stochastic* and
- a *deterministic* part.

13.4 Ionosphere Modeling

13.4.1 Deterministic Component

GPS-derived ionosphere models describing the *deterministic* component of the ionosphere usually are based on the so-called *single-layer* or *thin-shell* model outlined in Figure 13.3. This model assumes that all free electrons are concentrated in a shell of infinitesimal thickness. The *single-layer* mapping function F_I may be written using eqn. (13.2) as

$$F_I(z) = \frac{E}{E_\mathrm{v}} = \frac{1}{\cos z'} \qquad \text{with} \quad \sin z' = \frac{R}{R+H}\,\sin z, \tag{13.8}$$

where

z, z'	are the zenith distances at the height of the station and the single layer, respectively,
R	is the mean radius of the Earth, and
H	is the height of the single layer above the Earth's surface.

The height of this idealized layer is usually set to the height of the maximum electron density expected (see H_{max} in Figure 13.1). Furthermore the electron density E — the surface density of the layer — is assumed to be a function of geocentric latitude β and sun-fixed longitude s.

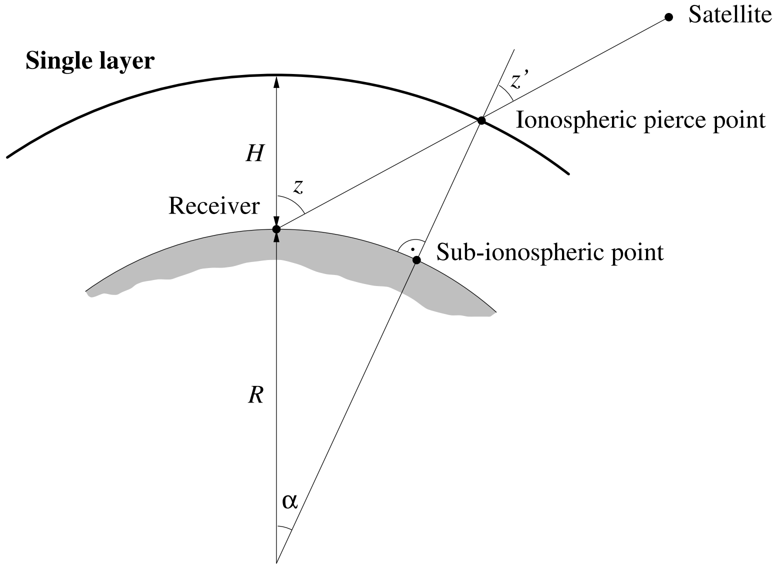


Figure 13.3: Single-layer model

To measure the TEC, the so-called *geometry-free* linear combination (9.24), which contains ionospheric information only (see Table 13.2), is analyzed. The particular observation equation for phase observations reads as

$$L_4 = -\alpha \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right) F_I(z)\, E(\beta, s) + B_4, \tag{13.9}$$

where

$L_4 = L_1 - L_2$ is the geometry-free phase observable (in meters),

$\alpha = 4.03 \cdot 10^{17}$ m s^{-2} TECU^{-1} is a constant,

f_1, f_2 are the frequencies associated with the carriers L_1 and L_2,

$F_I(z)$ is the mapping function evaluated at the zenith distance z',

$E(\beta, s)$ is the *vertical* TEC (in TECU) as a function of latitude β and sun-fixed longitude s, and

$B_4 = \lambda_1 N_1 - \lambda_2 N_2$ is a *constant* bias (in meters) due to the initial phase ambiguities N_1 and N_2 with their corresponding wavelengths λ_1 and λ_2; if a new ambiguity is set up for one satellite, a new parameter of this type has to be introduced.

The Bernese GPS Software Version 4.0 supports two types of ionosphere models to represent the *deterministic* component of the ionosphere:

(1) *local* models based on two-dimensional Taylor series expansions and

(2) *global* (or *regional*) models based on spherical harmonic expansions.

Note that the numbers enclosed in brackets correspond to the internally used model type numbers (see Figures 13.5 and 13.7).

13.4.1.1 Local TEC Model

The *local* TEC model — applicable in the vicinity of one or more dual-frequency station(s) — is represented by

$$E(\beta, s) = \sum_{n=0}^{n_{\max}} \sum_{m=0}^{m_{\max}} E_{nm} \, (\beta - \beta_0)^n \, (s - s_0)^m , \tag{13.10}$$

where

$n_{\max}, m_{\max}$ are the maximum degrees of the two-dimensional Taylor series expansion in latitude β and in longitude s,

E_{nm} are the (unknown) TEC coefficients of the Taylor series, i. e. the *local ionosphere model parameters* to be estimated, and

β_0, s_0 are the coordinates of the origin of the development.

β is the geocentric latitude of the intersection point of the line receiver–satellite with the ionospheric layer and s is the *sun-fixed* longitude of the ionospheric pierce point (or sub-ionospheric point), which is related to the local solar time (LT) according to

$$s = \mathrm{LT} - \pi \approx \mathrm{UT} + \lambda - \pi . \tag{13.11}$$

UT is Universal Time and λ denotes the geographical longitude of the sub-ionospheric point. For satellites at elevation angles of $15/20^{\circ}$ with widely different azimuth, these sub-ionospheric points can be separated by up to 3000/2000 km. Nevertheless, the representation (13.10) is *not* well suited for *regional* or *global* TEC models because of limitations in the (β, s)-space. More information concerning local ionosphere modeling may be found in [*Wild*, 1994].

13.4.1.2 Global TEC Model

The *global* TEC model — which may be used for regional applications also — may be written as

$$E(\beta, s) = \sum_{n=0}^{n_{\max}} \sum_{m=0}^{n} \widetilde{P}_{nm}(\sin\beta) \; (a_{nm} \, \cos ms + b_{nm} \, \sin ms) \, , \tag{13.12}$$

where

$n_{\max}$ is the maximum degree of the spherical harmonic expansion,

$\widetilde{P}_{nm} = \Lambda(n,m) \, P_{nm}$ are the *normalized* associated Legendre functions of degree n and order m based on normalization function $\Lambda(n,m)$ and Legendre functions P_{nm}, and

a_{nm}, b_{nm} are the (unknown) TEC coefficients of the spherical harmonics, i. e. *the global ionosphere model parameters* to be estimated.

Here we may use the geocentric latitude β and the sun-fixed longitude s or an equivalent set in the solar-geomagnetic system as independent arguments. Further information concerning global and regional ionosphere modeling may be found in [*Schaer et al.*, 1995] and [*Schaer et al.*, 1996].

Both ionosphere models represented by eqns. (13.10) and (13.12) do *not* provide a model for the time dependence in the *sun-fixed* reference frame because the "frozen" TEC structure is co-rotating with the Sun. However, there is always a time dependence in the *earth-fixed* frame.

13.4.2 Stochastic Component

Short-term TEC variations are *not* modeled by eqns. (13.10) and (13.12). They will be interpreted as noise of the geometry-free (L4) observable.

To model the *stochastic* component of the ionosphere, you have the possibility to set up the ionospheric term I of the double-difference observation equation (9.17) — rewritten in a simpler way —

$$L_1 = \varrho - I + \ldots + \lambda_1 N_1 \tag{13.13a}$$

$$L_2 = \varrho - \frac{f_1^2}{f_2^2} I + \ldots + \lambda_2 N_2 \tag{13.13b}$$

as an *unknown* parameter, called *Stochastic Ionosphere Parameter (SIP)*, representing the double-difference ionospheric delay on L1 according to (13.5). One SIP per epoch and satellite (or satellite pair) has to be estimated. To handle the huge number of SIP parameters, an *epoch-wise* parameter pre-elimination is performed.

This parameter type is particularly useful for "dual-band" ambiguity resolution when using strategies like the *General-Search* or the *Quasi-Ionosphere-Free (QIF)* strategy, which directly solve for the L1/L2 ambiguities (see also Chapter 15). In the *ambiguity-unresolved* case[2], you have to define a priori constraints on the SIP parameters to retain the *integer* nature of the L1/L2 ambiguities, otherwise you will implicitly get *real-valued* ambiguity parameters B_3 according to eqns. (9.21) and (9.22).

In addition, SIPs allow to continuously switch between a pure L1/L2 solution and an ionosphere-free (L3) solution which is demonstrated in Figure 13.4 for a 20-km baseline observed in the rapid-static scenario with a 5-satellite constellation [*Schaer*, 1994]. The formal accuracy of the coordinates/ambiguities is plotted with solid/dotted lines. We may recognize in Figure 13.4 that (a) there is a transitional zone where the a priori σ for the SIPs is of the order of the a priori measurement noise (4 mm), and (b) for a very large a priori σ, the 8 dotted curves showing the formal accuracy of the L1/L2 ambiguities go to infinity.

[2] Neither L1 and L2 ambiguities (N_1 and N_2) nor L5 ambiguities ($N_5 = N_1 - N_2$) are known.

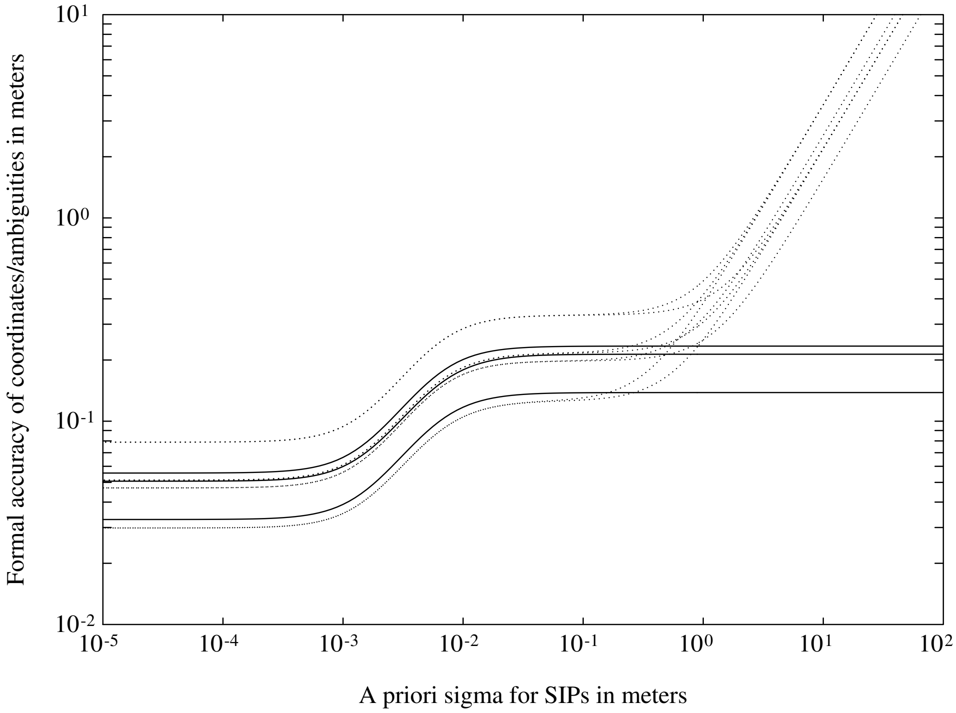


Figure 13.4: Coordinates and ambiguity parameters as a function of SIP constraining

13.5 Estimation of Deterministic Ionosphere Models

13.5.1 Local Ionosphere Models

Local ionosphere models (type-1 models) according to eqn. (13.10) may be estimated with program IONEST by activating Menu 4.7 .

```
4.7                    PROCESSING: IONOSPHERE MODEL

  CAMPAIGN         >         <     (blank for selection list)

 Input Files:
  MEASUREMENT TYPE > PHASE   <     (CODE, PHASE)
  OBSERVATIONS     >         <     (blank for selection list)
  COORDINATES      >         <     (blank for selection list)
  ECCENTRICITIES   > NO      <     (NO, if not used; blank for sel.list)
  STANDARD ORBIT   >         <     (blank for selection list)

 Output Files:
  IONOSPHERE MODEL > IONTST  <     (NO, if not to be saved)
  RESIDUALS        > NO      <     (NO, if not to be saved)
```

In Panel 4.7 you decide whether you want to analyze the *geometry-free* linear combination of either CODE or PHASE observations. We recommend to analyse PHASE observations. Under OBSERVATIONS one or more *zero-difference* observation files may be selected. If you want to derive more than one ionosphere model per session, you have to concatenate/split up the observation file(s) either on the RINEX or the Bernese-binary-format level into the sub-sessions requested (see Menu 2.5.6.1 or Menu 5.1), because program IONEST always takes *all* available observations. Furthermore you have to combine the individual ionosphere model files created into one common file (see example in Figure 13.5). For longer sessions (e. g. 24-hour sessions) it is much easier to generate a *regional* ionosphere model (see 13.5.2) than a *local* model. But, if you want to produce a TEC model based on *one* dual-frequency station only, you must use program IONEST, because program GPSEST, which in principle supports both model types, requires *single-difference* observation files. Please note that the estimation of *local* ionosphere models using program GPSEST is *not* recommended, because this possibility is *not* menu-supported[3].

The estimated ionosphere models are used in further processing steps, therefore you have to specify under `IONOSPHERE MODEL` an ionosphere model file name (e. g. `IONTST`). The ionosphere files (`*.ION`) are stored in the campaign-specific `ATM` directory. It is recommended to create a residual file (`*.RES`) containing L4 residuals is recommended, if you wish to study short-term TEC variations like scintillations or *Traveling Ionospheric Disturbances (TIDs)*. Use Panel 5.3.1 to browse through these files.

In Panel 4.7–1 you may define some preprocessing options to mark outliers when processing *code* measurements, or, to set up a new ambiguity parameter B_4 (according to observation equation (13.9)) for each cycle slip detected when processing *phase* measurements. The model-specific options include (a) `MIN. ELEVATION`, the minimum elevation to be processed, (b) `HEIGHT OF THE LAYER`, the single-layer height H (see mapping function (13.8) and Figure 13.3), (c) `DEGREE OF DEVELOPMENT IN LATITUDE` and `DEGREE OF DEVELOPMENT IN HOUR ANGLE`, $n_{\max}$ and $n_{\max}$ of TEC representation (13.10), and (d) `MAXIMUM DEGREE IN MIXED COEFFICIENTS`, the maximum allowed sum $(n + m)$ of both indices of the TEC parameters E_{nm} to be set up. Note that the values given in this panel are the recommended ones.

```
4.7-1                    IONOSPHERE MODEL: INPUT

  TITLE   >                                                    <

  Preprocessing:
    PRINT MESSAGES                      > NO  <    (YES or NO)
    CARRIER FOR BREAK DETECTION         > L4  <    (L3 or L4)
    POLYNOMIAL DEGREE                   > 1   <    (0,1,2,3)
    MAX.INTERVAL FOR TEST               > 4   <    minutes
    RMS OF ONE OBSERVATION             > 0.01  <   meters

  Processing Options:
    PRINT RESIDUALS                     > NO  <    (YES or NO)
    MIN. ELEVATION                      > 15  <    degrees
    HEIGHT OF THE LAYER                 > 400 <    kilometers
    DEGREE OF DEVELOPMENT IN LATITUDE   > 1   <
    DEGREE OF DEVELOPMENT IN HOUR ANGLE > 2   <
    MAXIMUM DEGREE IN MIXED COEFFICIENTS  > 2   <
```

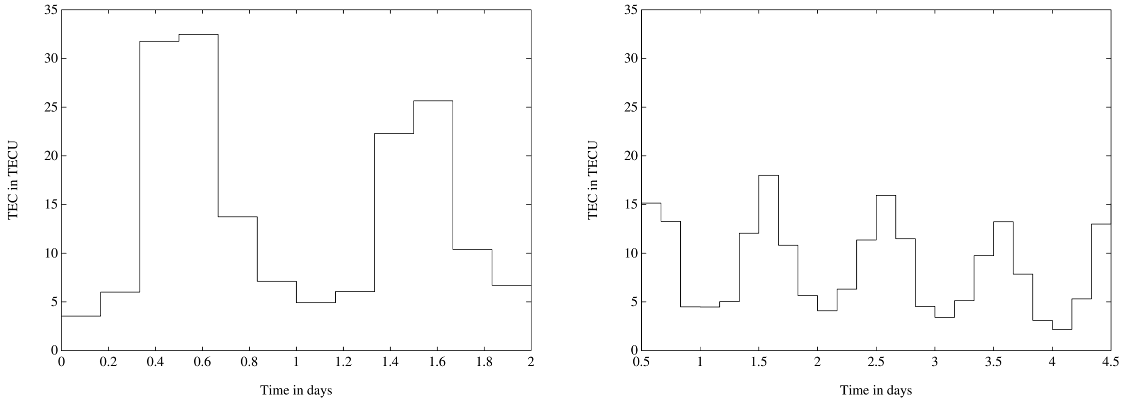
[3] In addition you have to prepare in a previous step the header of the ionosphere file to be introduced in program GPSEST.

```
IONOSPHERE MODELS FOR TURTMANN                                    8-FEB-93 10:59
--------------------------------------------------------------------------------
IONOSPHERE MODEL NUMBER                              :    1
TYPE OF IONOSPHERE MODEL                             :    1
ORIGIN OF DELEVOPMENT: TIME (UT)  (Y M D H)    : 1992 10 28 14.8
                       LATITUDE   (DEGREES)    :   46.8771
                       LONGITUDE  (DEGREES)    :    7.4651
                       HEIGHT OF LAYER (KM)    :  350
DEGREE OF DEVELOPMENT: TIME                    :    2
                       LATITUDE                :    1
                       MIXED                   :    2
NORMALIZATION FACTORS: LATITUDE (DEGREES)      :   6.00
                       TIME (HOURS)            :   2.00
                       ELECTRON CONTENT        :       0.10D+18
APPLICABILITY          FROM EPOCH              : 1992 10 28 12.0
                       TO EPOCH                : 1992 10 28 17.5
COEFFICIENTS:
DEG. LAT   DEG. TIME  COEFFICIENT       RMS
    0        0        0.26313868E+01   0.18961230E-01
    0        1       -0.11226929E+01   0.82974723E-02
    0        2        0.90513909E-02   0.10480726E-01
    1        0       -0.53071964E+00   0.10746679E-01
    1        1        0.88148393E-01   0.15985126E-01
   -1
IONOSPHERE MODEL NUMBER                              :    2
TYPE OF IONOSPHERE MODEL                             :    1
ORIGIN OF DELEVOPMENT: TIME (UT)  (Y M D H)    : 1992 10 29 14.8
                       LATITUDE   (DEGREES)    :   46.8771
                       LONGITUDE  (DEGREES)    :    7.4651
                       HEIGHT OF LAYER (KM)    :  350
DEGREE OF DEVELOPMENT: TIME                    :    2
                       LATITUDE                :    1
                       MIXED                   :    2
NORMALIZATION FACTORS: LATITUDE (DEGREES)      :   6.00
                       TIME (HOURS)            :   2.00
                       ELECTRON CONTENT        :       0.10D+18
APPLICABILITY          FROM EPOCH              : 1992 10 29 12.0
                       TO EPOCH                : 1992 10 29 17.5
COEFFICIENTS:
DEG. LAT   DEG. TIME  COEFFICIENT       RMS
    0        0        0.25439429E+01   0.11467723E-01
    0        1       -0.40731147E+00   0.50130496E-02
    0        2       -0.69612034E-01   0.64961719E-02
    1        0       -0.25940186E+00   0.64259418E-02
    1        1        0.48364446E+00   0.10364515E-01
   -1
```

Figure 13.5: Example of an ionosphere file containing (two) local TEC models

Figure 13.5 gives an example of an `ION` file containing a two-session model. When joining individual models, you have to guarantee that all models end with "`-1`" and *additional* models directly start with "`IONOSPHERE MODEL NUMBER`", i. e. *without* title lines.

A series of zero-degree TEC parameters E_{00} extracted from local ionosphere models is plotted in Figure 13.6. These parameters roughly describe the TEC over the reference station(s) processed in program IONEST. In this case the phase data of the permanent IGS station Zimmerwald has been used to estimate 4-hour ionosphere models, which were then taken into account when processing the 3-dimensional GPS test network in Turtmann, Switzerland [*Beutler et al.*, 1995]. In both subfigures you see the typical *diurnal* variation of the TEC, which is dominated by the "bulge" co-rotating with the Sun. The ionospheric conditions may vary considerably as visualized by the plots drawn with the same scale (compare also Figure 13.2).



(a) 1992 Turtmann campaign

(b) 1993 Turtmann campaign

Figure 13.6: Zero-degree TEC parameters E_{00} extracted from local ionosphere models

13.5.2 Global/Regional Ionosphere Models

The estimation of *global/regional* ionosphere models (or maps) is supported by the main parameter estimation program GPSEST, which may be started with Menu 4.5. Note that the program GPSEST implicitly processes *double-difference* GPS observations, as opposed to the program IONEST, which processes *zero-difference* observations. In the following paragraph we discuss the most important options to be specified to estimate global/regional ionosphere model parameters (GIM parameters).

In Panel 4.5 we recommend to select single-difference *phase* file(s) at `PHASE S.DIFF.`.

To save the estimated ionosphere models for further processing steps, you have to specify in Panel 4.5–0 under `IONOSPHERE MODELS` an ionosphere model file name. It is *not* recommended to enter an IONEX file name under `IONOSPHERE MAPS` because this file type is used for test purposes only. Both ionosphere-specific files (`*.ION` and `*.INX`) are stored in the campaign-specific `ATM` directory.

In Panel 4.5–1 two parameters are essential: (a) the frequency to be analyzed (option `FREQUENCY`), `L4` is recommended and (b) in the input field `STATION`, the selection of the station(s) to be fixed, where `ALL` is mandatory.

The setting-up of "special" parameter types[4] has to be initiated in Panel 4.5–2 with `YES` under `SPECIAL REQUESTS`. You will get Panel 4.5–2.4, where you have to select `COE` under `GLOBAL IONOSPHERE MODEL PARAMETERS` to set up GIM parameters.

[4] Coordinates, ambiguities, and orbit parameters do *not* fall into this category.

```
4.5-2.4              PARAMETER ESTIMATION: SPECIAL REQUESTS

  Special Requests:

    A PRIORI SIGMAS FOR SITE COORDINATES        > NO   <   (YES.. NO)

    SITE-SPECIFIC TROPOSPHERE PARAMETERS        > NO   <   (YES.. NO)
    STOCHASTIC IONOSPHERE PARAMETERS            > NO   <   (YES.. NO)
    GLOBAL IONOSPHERE MODEL PARAMETERS          > COE  <   (COE.. HGT.. NO)
    EARTH ROTATION PARAMETERS                   > NO   <   (YES.. NO)
    COORDINATES OF CENTER OF MASS               > NO   <   (YES.. NO, ASIS)
    SATELLITE ANTENNA OFFSETS                   > NO   <   (YES.. NO)
    RECEIVER CLOCK ERRORS                       > NO   <   (YES.. NO)

    PARAMETER PRE-ELIMINATION                   > NO   <   (YES.. NO, ASIS)

    SATELLITE-SPECIFIC A PRIORI SIGMAS          > NO   <   (YES.. NO)
```

In Panel 4.5–2.4.C, you may enter your model-specific requests. The `NUMBER OF COEFFICIENT SETS PER SESSION` should be set to `1` for *regional* applications assuming a (maximum) session length of 24 hours. A higher number of coefficient sets (models) might be appropriate for *global* application.

```
4.5-2.4.C      PARAMETER ESTIMATION: GLOBAL IONOSPHERE MODEL PARAMETERS

   Number of Ionosphere Models and Coefficients:
     NUMBER OF COEFFICIENT SETS PER SESSION  >  1 <
     MAXIMUM DEGREE OF SPHERICAL HARMONICS   >  8 <
     MAXIMUM ORDER  OF SPHERICAL HARMONICS   >  8 <

   Modeling Characteristics:
     TIME-DEPENDENCY            > STATIC  <      (STATIC or DYNAMIC)
     SUN-FIXED REFERENCE FRAME  > GEOGRAPHIC  <  (GEOGRAPHIC or GEOMAGNETIC)
     LONGITUDE OF THE SUN       > MEAN <         (MEAN or TRUE)
     MAPPING FUNCTION           > COSZ <         (COSZ)

   Additional Information:
     A PRIORI HEIGHT OF SINGLE LAYER  >  400.00 <  km
     LATITUDE  OF GEOMAGNETIC POLE    >   79.00 <  degrees
     LONGITUDE OF GEOMAGNETIC POLE    >  -71.00 <  degrees
     A PRIORI SIGMA FOR COEFFICIENTS  >    0.00 <  TECU     (0: no sigma)
```

`MAXIMUM DEGREE OF SPHERICAL HARMONICS` and `MAXIMUM ORDER OF SPHERICAL HARMONICS` correspond to $n_{\max}$ and $m_{\max}(\leq n_{\max})$ of TEC model (13.12). For *regional* models a lower maximum degree than given in the above panel should be specified (e. g. $n_{\max} = 5, m_{\max}+5$), depending on the extent of the network processed. Assuming $m_{\max} = n_{\max}$, $(n_{\max} + 1)^2$ GIM parameters per session will be estimated.

For `TIME-DEPENDENCY` you may select either `STATIC` to create ionosphere models representing *static* (or "frozen") TEC structures in the sun-fixed reference frame which are referring to specific time intervals or `DYNAMIC` to model the TEC coefficients as piece-wise linear functions $a_{nm}(t)$ and $b_{nm}(t)$ representing a (low-)*dynamic* ionosphere $E(\beta, s, t)$. If you select `DYNAMIC`, the TEC coefficients are always related to particular reference epochs.

With the option `SUN-FIXED REFERENCE FRAME` you may decide in which reference frame the TEC should be modeled, a `GEOGRAPHIC` or a `GEOMAGNETIC` frame.

With the setting `MEAN/TRUE` for the `LONGITUDE OF THE SUN`, the argument s is computed according to the right- *or* left-hand side of eqn. (13.11).

It is recommended to set the `A PRIORI HEIGHT OF SINGLE LAYER` to $H = 400$ km.
In the fields `LATITUDE OF GEOMAGNETIC POLE` and `LONGITUDE OF GEOMAGNETIC POLE` you have to enter the coordinates of the earth-centered dipole axis[5], if you select `GEOMAGNETIC` for the option `SUN-FIXED REFERENCE FRAME`.
We recommend to introduce weak a priori weights (e. g. 10 TECU) in the field `A PRIORI SIGMA FOR COEFFICIENTS` when producing *regional* models.

By selecting `HGT` in the field `GLOBAL IONOSPHERE MODEL PARAMETERS` in Panel 4.5–2.4 , you may set up additional single-layer height parameters as *unknown* parameters. In that case GPSEST requires an a priori GIM file — stemming from an initial program run — to be specified in Panel 4.5 (field `IONOSP. MODELS`), because the parameter estimation problem is no longer *linear*. Panel 4.5–2.4.C is automatically skipped and you will immediately see Panel 4.5–2.4.D. Please note that this option has been designed for *global* applications only.

```
4.5-2.4.D          PARAMETER ESTIMATION: HEIGHT OF SINGLE LAYER

   Number of Single-Layer Height Parameters:
     NUMBER OF HEIGHT PARAMETERS  > ALL  <  (one for ALL  ionosphere models,
                                             one for EACH ionosphere model)

   A Priori Sigma for Height Parameters:
     A PRIORI SIGMA FOR HEIGHT PARAMETERS  >    0.00 <  km  (0: no sigma)
```

Using Menu 5.6.5 you may extract — among other items — GIM-specific information from GPSEST output files by entering a file name for the `GIM SUMMARY` in Panel 5.6.5. All `SUM` files are stored in the `OUT` directory.

Figure 13.7 shows an example of an `ION` file containing a global one-day model. To join a series of global/regional models (type-2 models) stored in individual `ION` files into a “multi-session” model, you may simply copy these files together in *chronological* order.

The GIM listed in Figure 13.7 is visualized in Figure 13.8. The typical “bulge” aligned with the Sun ($s \approx 0$) can easily be seen. The latitude band covered is indicated by two dashed lines.

[5] Coordinates of the geomagnetic pole can be obtained by telneting to `neis.cr.usgs.gov` (`136.177.20.9`) with the user name `QED`.

```
CODE: GLOBAL IONOSPHERE MODEL FOR DAY 073, 1996                  17-MAR-96 04:30
--------------------------------------------------------------------------------
IONOSPHERE MODEL NUMBER                          : 0731-00
TYPE OF IONOSPHERE MODEL (1=LOCAL,2=GLOBAL)      :    2
MAXIMUM DEGREE OF SPHERICAL HARMONICS            :    8
MAXIMUM ORDER                                    :    8
DEVELOPMENT WITH RESPECT TO
  GEOGRAPHICAL (=1) OR GEOMAGNETIC (=2) FRAME    :    1
  MEAN (=1) OR TRUE (=2) POSITION OF THE SUN     :    1
MAPPING FUNCTION (1=1/COS)                       :    1
HEIGHT OF SINGLE LAYER AND ITS RMS ERROR (KM)    :  400.00     0.00
COORDINATES OF EARTH-CENTERED DIPOLE AXIS
  LATITUDE OF NORTH GEOMAGNETIC POLE (DEGREES) :
  EAST LONGITUDE (DEGREES)                       :
PERIOD OF VALIDITY
  FROM EPOCH / REFERENCE EPOCH (Y,M,D,H,M,S)     : 1996 03 13 00 00 00
  TO EPOCH                                       : 1996 03 13 23 59 59
LATITUDE BAND COVERED
  MINIMUM LATITUDE (DEGREES)                     :  -85.75
  MAXIMUM LATITUDE (DEGREES)                     :   87.09
ADDITIONAL INFORMATION
  NUMBER OF CONTRIBUTING STATIONS                :   64
  TID INDICATOR (TECU)                           :    0.99
COMMENT / WARNING                                :
COEFFICIENTS
DEGREE  ORDER    VALUE (TECU)   RMS (TECU)
   0       0       10.66643935     0.0322
   1       0       -0.98758858     0.0310
   1       1        4.64559206     0.0307
   .       .        ..........     ......
   .       .        ..........     ......
   .       .        ..........     ......
   8      -8        0.06239174     0.0105
```

Figure 13.7: Example for an ionosphere file containing a global TEC model

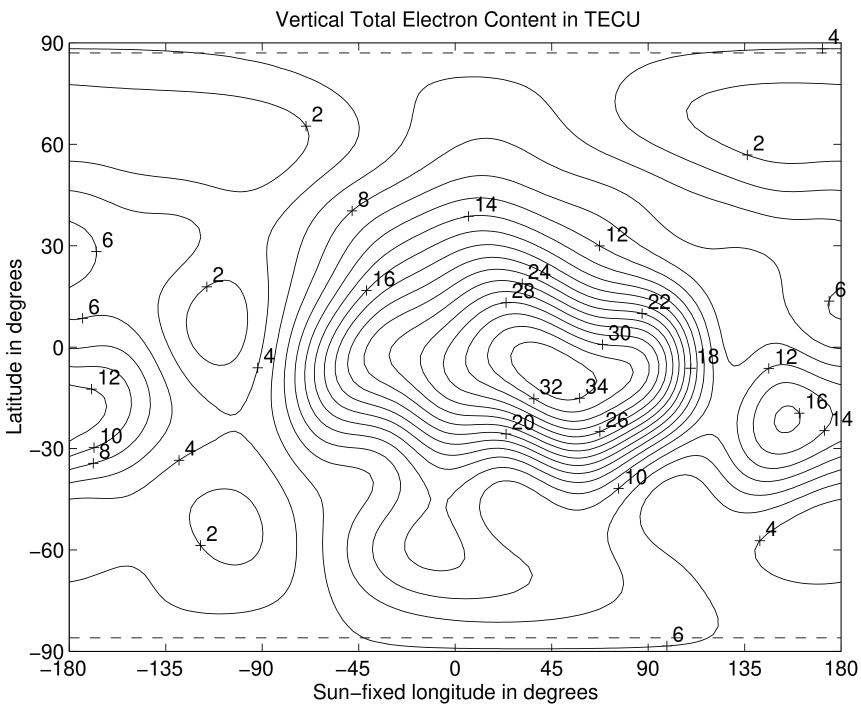


Figure 13.8: GIM for day 073, 1996

Since 1 January 1996, the CODE Analysis Center routinely produces daily global ionosphere models (GIMs) as an additional product. Apart from that, GIMs for the entire year 1995 have been computed in a re-processing step [*Schaer et al.*, 1996]. The corresponding `ION` files starting with day 001, 1995, are available via anonymous ftp (see also Chapter 7). Regional ionosphere models for *Europe* — routinely generated since December 1995 — are available on special request.

Figure 13.9 shows the *mean* TEC, which has been extracted from the daily GIMs produced by CODE. This parameter roughly describes the ionospheric activity on a global scale (compare also Figure 13.2).

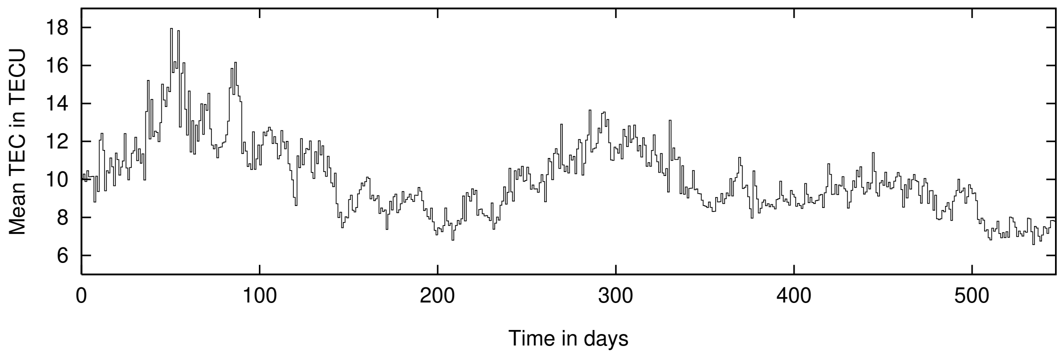


Figure 13.9: 1.5-year GIM statistics from 1 January 1995 to 30 June 1996

13.5.3 Application of Deterministic TEC Models

Deterministic TEC models may be used by two programs:

- the *pre-processing* program MAUPRP and
- the *parameter estimation* program GPSEST.

The requested `ION` file has to be specified in the option field `IONOSP. MODELS` in Panel 4.4.2 and Panel 4.5, respectively. Both programs will automatically detect whether *local* (type-1) or *global/regional* (type-2) ionosphere models are introduced.
Note that the *single-positioning* program CODSPP only supports a very simple ionosphere model with "hard-wired" values for the day- and night-time electron content which is therefore *not* really representative for actual ionospheric conditions.

Where can *deterministic* ionosphere models help in the GPS data processing?

- In the *pre-processing* when large TEC gradients occur. But note, that short-term TEC variations are *not* reflected in the deterministic ionosphere models, i. e. strong scintillations will still harm pre-processing.
- For the *ambiguity resolution* to make the ambiguity fixing more reliable by reducing the fractional parts of (wide-lane) ambiguities, if you do *not* use (precise) dual-band code measurements by analyzing the Melbourne-Wübbena linear combination (9.28).

- In the *parameter estimation step* to reduce the ionosphere-induced scale bias in the GPS network solution (see Table 13.1), if you process L1 and/or L2 observations and *not* the *ionosphere-free* linear combination (L3).

13.6 Stochastic Ionosphere Modeling Technique

13.6.1 Estimation of Stochastic Ionosphere Parameters

Stochastic Ionosphere Parameters (SIPs), the terms I in eqn. (13.13), may be set up in Panel 4.5–2.4 (see `STOCHASTIC IONOSPHERE PARAMETERS`). In Panel 4.5–2.4.7, you may specify then several options concerning the SIPs.

```
4.5-2.4.7     PARAMETER ESTIMATION: STOCHASTIC IONOSPHERE PARAMETERS

  Stochastic Ionosphere Parameters:

    EPOCH-WISE PRE-ELIMINATION                            > YES <   (YES,NO)
    ELIMINATION OF REFERENCE IONOSPHERE PARAMETERS        > YES <   (YES,NO)

    ABSOLUTE A PRIORI SIGMA ON SINGLE DIFFERENCE LEVEL  > 0.25   < m
    RELATIVE A PRIORI SIGMA OF IONOSPHERIC RANDOM WALK  > 0.00   < m/min**1/2
```

With `EPOCH-WISE PRE-ELIMINATION` a special parameter pre-elimination algorithm working epoch by epoch may be activated. This is a recommended procedure because of the huge number of SIPs usually set up. Note that the *epoch-wise* parameter pre-elimination may be enforced in Panel 4.5–2.4.8 with `EP` at `DIFF. IONOSPHERE`, too.

`ELIMINATION OF REFERENCE IONOSPHERE PARAMETERS` is the option where you may decide whether you want to estimate SIPs on the *double-difference* or a *quasi-single-difference* level. The estimation on the quasi-single-difference level should be used when defining so-called *relative* a priori σ in `RELATIVE A PRIORI SIGMA OF IONOSPHERIC RANDOM WALK`. If you eliminate *reference ionosphere parameters*, the resulting SIPs are estimated with respect to a *reference satellite*, that satellite actually, which is *closest* to the *zenith*.

An *absolute* a priori σ must be specified in the field `ABSOLUTE A PRIORI SIGMA ON SINGLE DIFFERENCE LEVEL` to get “hybrid” dual-band observations. By entering “0.00” *no* SIP constraints are introduced. When using the *General-Search* ambiguity resolution strategy in combination with the stochastic ionosphere modeling, we recommend to specify absolute a priori σ between — let's say — 0.01 m and 0.1 m, and between 0.1 m and 1 m when using the *Quasi-Ionosphere-Free (QIF)* strategy (see also Figure 13.4).

Relative a priori constraints between consecutive SIPs of the same satellite may be defined to model the correlation in time of the ionospheric signal. This option may be used only if you do *not* eliminate *reference ionosphere parameters* (option `ELIMINATION OF REFERENCE IONOSPHERE PARAMETERS`). This “SIP smoothing” might be useful e. g. for *kinematic* applications under quiet ionospheric conditions.

Figure 13.10 shows the resulting SIPs for a European 600-km baseline of the IGS network. The approximately 12 000 parameters which describe the *differential* ionospheric delay on L1 have been estimated in several program runs by defining time windows. Short-term variations like so-called

Medium-Scale Traveling Ionospheric Disturbances (MSTIDs) with their typical periods of 10 to 60 minutes may be recognized.

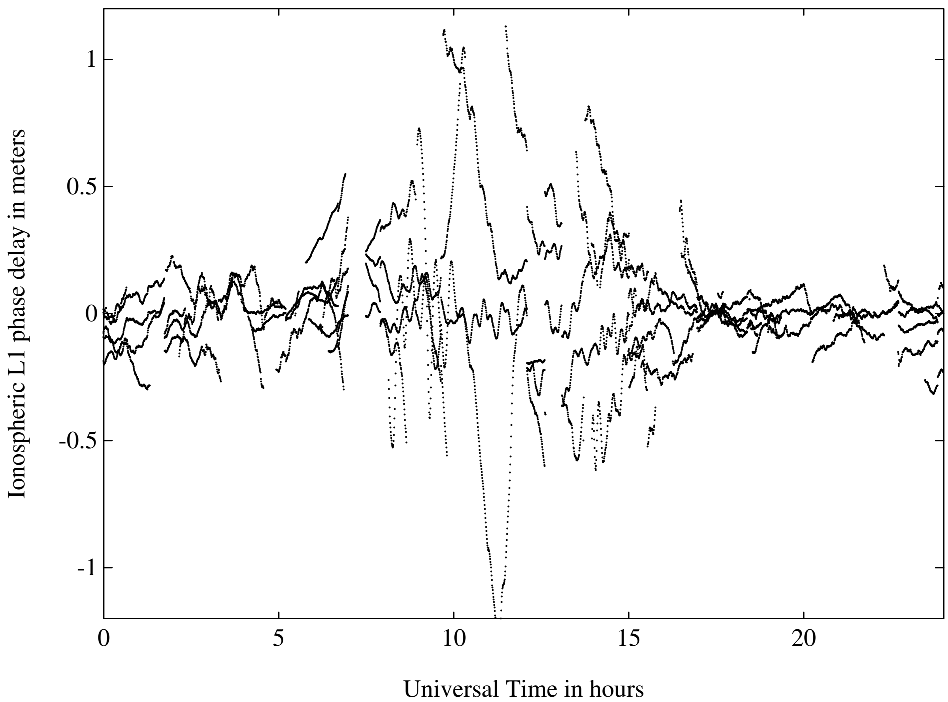


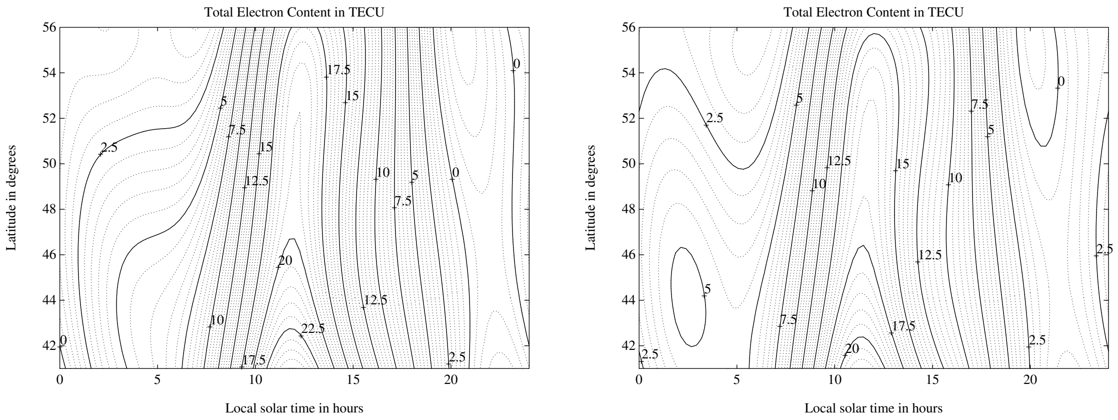
Figure 13.10: Stochastic ionosphere parameters (SIPs) describing the double-difference ionospheric delay on L1

13.6.2 Using Stochastic Ionosphere Parameters

The main application for *stochastic ionosphere modeling* is *ambiguity resolution* using strategies like the *General-Search* and the *QIF* strategy, both directly solving for the L1/L2 ambiguities. There is another possible use as already demonstrated in Figure 13.4: By varying the a priori constraints on the SIP parameters, you have the possibility to continuously switch between a pure L1/L2 solution and an L3 solution.

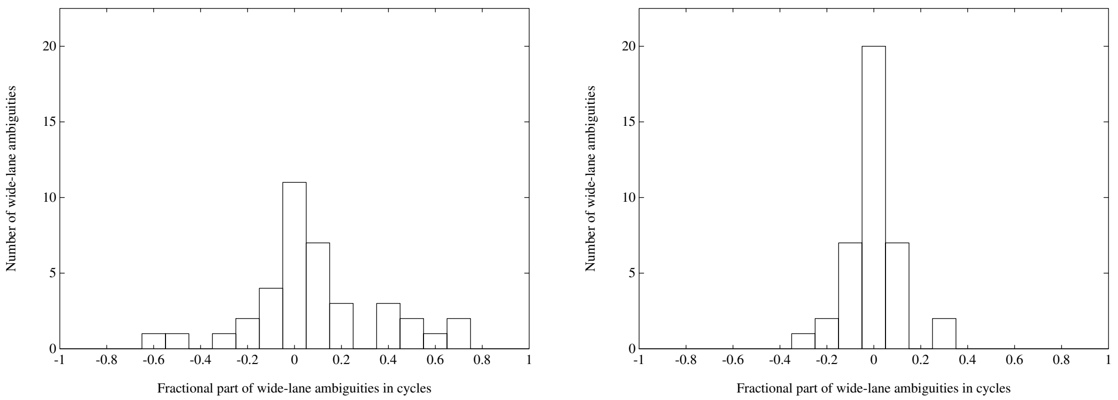
Finally, we have to emphasize that "hybrid" dual-band observations contain in principle *all* information concerning geometry *and* ionosphere. Therefore it is allowed to set up GIM parameters in addition to SIP parameters to instantaneously separate — in a single processing run — the *stochastic* and the *deterministic* component.

Figure 13.11 shows a regional ionosphere model as derived from doubly differenced phase data of *one* baseline (a) *before* and (b) *after* the QIF ambiguity resolution. Large values and rms errors for the *regional* TEC parameters often occur due to the limited latitude range covered and may be ignored — as in this example. The resulting "fractional parts" of the wide-lane ambiguities are shown in Figure 13.12, when (a) *no* deterministic TEC parameters are set up and (b) GIM parameters are estimated.



(a) *Before* ambiguity resolution (b) *After* ambiguity resolution

Figure 13.11: Regional (or baseline-specific) ionosphere model



(a) *Without* IM (b) *With* IM

Figure 13.12: Fractional parts of wide-lane ambiguities indicating the (remaining) *deterministic* part of the ionosphere