14. Earth Orientation Modeling and Estimation

14.1 Motivation

By the term *Earth Orientation Parameter* (EOP) we understand a set of *5 parameters* describing the orientation of the ITRF *(International Terrestrial Reference Frame)* in the ICRF *(International Celestial Reference Frame)* in conjunction with the conventional Precession-Nutation model. The parameters x_p, y_p locate the *Celestial Ephemeris Pole* (CEP) in the terrestrial reference frame, whereas the position of the CEP in inertial space is defined by the IAU 1980 Theory of Nutation. The parameters $d\psi$ and $d\varepsilon$ reflect the difference between the actual position of the celestial pole and the position given by the conventional IAU-model. Finally the difference UT1–UTC (respectively UT1–TAI, TAI=International Atomic Time) gives access to the direction of the IERS Reference Meridian in the Celestial Reference Frame (for details we refer to [*Castrique*, 1996]).

Station positions are conveniently represented in an Earth-fixed reference frame rotating in a well-defined way with the Earth. This reference frame is realized today by a catalogue of adopted geocentric coordinates and a velocity field of tracking stations derived from the analysis of data from various space techniques (VLBI, SLR, GPS, DORIS). The frame is designated as the *International Terrestrial Reference Frame (ITRF)* maintained by the International Earth Rotation Service (IERS). The latest realization is called ITRF94 with positions given at epoch $t_0 = 1993.0$ [*Boucher et al.*, 1996].

The motion of the GPS satellites is best described in the *Inertial Celestial Reference Frame (ICRF)*, which is materialized by a list of adopted equatorial coordinates (epoch J2000.0) of extragalactic radio sources uniformly distributed over the sky.

In order to compute the difference vector between the observing station and the satellite both positions must be given in the same reference frame. Therefore, we need to know the current set of transformation parameters (EOP) between the two frames when analyzing GPS data. Moreover, when processing data from a global GPS tracking network, it is possible to estimate a subset of EOP parameters on condition that the positions of some tracking sites are known in the ITRF.

14.2 Theory

The transformation between the Earth-fixed and the celestial coordinate system may be performed by means of equation

$$\tilde{\mathbf{r}} = \mathbf{X}^T\ \mathbf{Y}^T\ \mathbf{U}\ \mathbf{N}\ \mathbf{P}\ \mathbf{r} \tag{14.1}$$

where $\tilde{\mathbf{r}}$ and $\mathbf{r}$ denote the Cartesian coordinates (column matrices) of a station in the terrestrial and inertial systems, respectively. The sequence of rotation matrices $\mathbf{N}\ \mathbf{P}$ describes the transformation between the mean celestial system at epoch J2000.0 and a system defined by the true equator and equinox of date. $\mathbf{P}$ and $\mathbf{N}$ may be written as follows

$$\begin{aligned}
\mathbf{P} &= \mathbf{R}_3(-z)\ \mathbf{R}_2(\theta)\ \mathbf{R}_3(-\zeta) && \text{(14.2a)}\\
\mathbf{N} &= \mathbf{R}_1(-\varepsilon_0 - \Delta\varepsilon)\ \mathbf{R}_3(-\Delta\psi)\ \mathbf{R}_1(\varepsilon_0) \approx \\
&\approx \mathbf{R}_1(-\Delta\varepsilon)\ \mathbf{R}_2(\Delta\psi \sin\varepsilon_0)\ \mathbf{R}_3(-\Delta\psi\cos\varepsilon_0) && \text{(14.2b)}
\end{aligned}$$

where

$\mathbf{R}_i(\alpha)$	...	characterizes a rotation around axis i and about angle α,
z, θ, ζ	...	are the precession parameters,
ε_0	...	denotes the mean obliquity of the ecliptic, and
$\Delta\psi, \Delta\varepsilon$	...	denote the nutation in longitude and obliquity respectively.

$\mathbf{U} = \mathbf{R}_3(\Theta_{GA})$ provides the transition to the rotating system where Θ_{GA} is the Greenwich apparent sidereal time. Finally the polar motion matrices

$$\mathbf{X} = \mathbf{R}_2(x_p)\ , \quad \mathbf{Y} = \mathbf{R}_1(y_p) \tag{14.3}$$

describe the separation between the third axis of the terrestrial system (the Conventional Terrestrial Pole = CTP) and the Celestial Ephemeris Pole.

To the accuracy level required *for the computation of the partial derivatives* of the GPS observable with respect to the parameters of interest, we may approximate the nutation matrix as a product of three infinitesimal rotations as shown in (14.2).

$$\mathbf{N} \approx \mathbf{R}_1(-\Delta\varepsilon)\ \mathbf{R}_2(\Delta\psi\sin\varepsilon_0)\ \mathbf{R}_3(-\Delta\psi\cos\varepsilon_0)$$

Introducing this result into equation (14.1) we obtain the following simplified transformation equation:

$$\mathbf{r} = \mathbf{P}^T(t)\ \mathbf{R}_1(\Delta\varepsilon)\ \mathbf{R}_2(-\Delta\psi\sin\varepsilon_0)\ \mathbf{R}_3(-\Theta_{GM})\ \mathbf{R}_1(y_p)\ \mathbf{R}_2(x_p)\ \tilde{\mathbf{r}}\ , \tag{14.4}$$

where Θ_{GM} stands for the Greenwich *mean* sidereal time. All Earth Orientation Parameters are contained in equation (14.4). It is our goal now to derive expressions for the partial derivatives of the GPS observable with respect to these parameters. Neglecting refraction effects and leaving out range biases, we essentially observe the slant range ϱ between the receiver position $\mathbf{r}_k$ at observation time t and the GPS satellite position $\mathbf{r}^i$ at time $t - \varrho/c$ where c stands for the velocity of light. The slant range may e.g. be computed in the celestial reference frame:

$$\varrho = \sqrt{(\mathbf{r}^i - \mathbf{r}_k)^T\ (\mathbf{r}^i - \mathbf{r}_k)}\ . \tag{14.5}$$

Let p denote one of the Earth Orientation Parameters (e.g. a polar wobble component). From equation (14.5), assuming that the partial derivative of the satellite position with respect to p is zero, we conclude

$$\frac{\partial \varrho}{\partial p} = -\mathbf{e}^T \, \frac{\partial \mathbf{r}_k}{\partial p} \tag{14.6}$$

where

$$\mathbf{e} = \frac{\mathbf{r}^i - \mathbf{r}_k}{\varrho} \tag{14.7}$$

is the unit vector (more precisely: Cartesian components of the vector in the ICRF) pointing from the receiver to the satellite. For more information we refer to [*Kleusberg and Teunissen*, 1996].

Unfortunately, due to correlations with the orbital elements, a subset of the EOP is not directly accessible to the GPS (namely ΔUT and the nutation parameters). This statement can easily be verified e.g. for ΔUT by comparing the partial derivative of ϱ (see eqn. (14.5)) with respect to the right ascension of the ascending node.

On the other hand, *it is possible* to solve for a drift in $UT1 - UTC$ by adopting a model of the following type:

$$\Delta UT = \Delta UT_{t=t_0} + \frac{\mathrm{d}}{\mathrm{d}\,t}\Big(\Delta UT\Big)_{t=t_0} \cdot (t - t_0) \tag{14.8}$$

Thanks to this time dependence the length of day (LOD) may very well be estimated with the GPS.

14.3 Use of Earth Orientation Parameters in the Bernese GPS Software

14.3.1 General Dataset Names

As mentioned previously a set of EOP is required to perform the transformation from the celestial to the Earth-fixed system or vice versa. The programs PRETAB and BRDTAB (Menu 3.2) compute e.g. a table of satellite positions in the inertial frame from the available orbit information (usually Precise Orbits in an Earth-fixed frame e.g. ITRF94). Therefore the EOP file corresponding to the orbits used has to be specified in Panel 0.3.1 (Pole Information). If IGS precise orbits are used the corresponding EOP files (covering always one GPS week) are available at the global data centers. For example the "Crustal Dynamics Data Information System (CDDIS)"

```
ftp  cddis.gsfc.nasa.gov    (128.183.10.141)
userid: anonymous
passwd: <ident>
cd  gps3:[products]
```

stores, besides the Precise Ephemeris files, pole information stemming from different analysis centers. The Earth Rotation Parameters associated with the IGS ephemeris files are computed using a weighted average of all available center-specific ERP files. (The term Earth Rotation Parameters (ERP) is used for a 3-parameter subset of the EOP which comprises polar motion (x_p, y_p) and $UT1$.) The ERP-files stemming from CODE may be downloaded from our anonymous ftp (ubeclu.unibe.ch, directory AFTP:[CODE]), too. Be aware that in contrast to the naming rules of the Bernese environment all these files contain pole information in the IERS format although their extension is ".ERP" (see also Chapter 23)! The pole files in IERS format have to be transformed to the Bernese pole format. This task is performed by program POLUPD (Menu 5.5.1) with options to be discussed in detail later.

If other than IGS precise orbits are used (and no specific ERP files are available), it is recommended to use the IERS C04 pole. C04 is a continuous, slightly low pass filtered series at daily intervals and can be interpolated linearly. Oscillations in UT1 and in the length of day due to zonal tides for periods under 35 days, as well as the 14-day terms in x_p and y_p are present in full size in the series [*Castrique*, 1996].

The EOP series mentioned above may be obtained via anonymous FTP. Data files concerning Earth rotation (C04 series, IERS Bulletin B) and celestial frame are stored at the IERS Central Bureau and may be picked up as follows:

```
ftp  hpvlbi.obspm.fr        (145.238.100.7)
userid: anonymous
passwd: "e-mail address"
cd /iers
```

Quick-look EOP series (IERS Bulletin A) can be transferred from:

```
ftp maia.usno.navy.mil        (192.5.41.22)
userid: anonymous
passwd: <ident>
cd  ser7
```

Information concerning updates of internet addresses and server names is always provided by the most recent IERS Annual Report. IERS pole files already transformed into the Bernese format may be downloaded from our anonymous ftp, too. They are located in the directory AIUB$FTP:[BSWUSER.GEN] (files C04_*.ERP, RAP_*.ERP, see also Chapter 7).

14.3.2 Update of Pole Information

In order to obtain the most accurate EOP information covering the span of your GPS campaign you are forced to update the file containing the Earth Rotation Parameter information, whenever more consistent (with the "Precise Orbit" data) and/or more recent values become available. The best way to perform this task is by means of the service programs POLUPD (Menu 5.5.1) and POLXTR (Menu 5.5.2).

POLUPD takes information from input files of (almost) any conventional format and converts the data to the Bernese pole format. Usually the extension ".IEP" indicates a file containing pole information in a foreign format, the extension ".ERP" denotes Bernese format. You have to keep this in mind whenever you leave input fields blank in Panel 5.5.1 and the menu system shows you a list of available input files.

Program POLUPD requires two files that have to be located in the `X:[GEN]` directory. Their names have to be specified in Panel 0.3.1. The first file contains a table of all leap seconds during the last several years (see variable "`LEAP SECOND`" in Menu 0.3.1 and Chapter 23). You have to modify this file whenever a new leap second is announced by the IERS. The second file is the so-called Pole Offset Coefficient file. It contains values which have to be added to the EOP(IERS) time series in order to make them consistent with the current realization of the IERS terrestrial and celestial reference frames (see Chapter 23).

The example below illustrates the use of the subsequent Panel 5.5.1–1. The option `NUTATION MODEL` specifies the type of model you use to apply the nutation correction parameters. There are three different entries. The `HERRING` model is a model derived from VLBI observations, `OBSERVED` specifies that nutation corrections given in columns `DEPS` and `DPSI` of the ERP file (to be created here) will be used. These corrections may stem from VLBI observations (e.g. in IERS Bulletin A) or from GPS observations computed by means of the Bernese software (research purpose). The recommended entry is NO. This choice prevents the use of nutation correction information and therefore the computations are solely based on the IAU 1980 nutation model.

```
5.5.1-1              POLE: UPDATE POLE FILE: GENERAL OPTIONS

  Header information:
    TITLE    >                                                              <
    NUTATION MODEL            > NO       <   (NO, OBSERVED, HERRING)

  Bulletin B as input:
    USE 1 OR 5 DAY VALUES     > 1        <   (1: one day values,
                                              5: five day values)
  Pole offsets:
    USE POLE OFFSET FILE        > YES <      (YES: Automatic,
                                              NO: add no pole offset)
  Window:
    USE WINDOW                  > NO  <      (YES,NO)

    FROM/TO              yyyy mm dd hh.hh      yyyy mm dd hh.hh
                         ---- -- -- -----      ---- -- -- -----
                       > 1993 01 01 00.00 <  > 1995 01 01 00.00 <
```

Program POLXTR (Menu 5.5.2) extracts ERP information from a list of consecutive pole files in the IERS format and allows you to specify an a priori pole file which may be used for comparison purposes. We refer to the corresponding help panel for details.

14.4 Estimation of Earth Orientation Parameters

Today, in view of the products made available through the IERS and the IGS it will in general *not* be necessary to solve for EOPs, but to use EOPs consistent with the orbits and coordinates used. For research type applications based on data stemming from a *global GPS network* it makes sense to set up EOPs. This section is important for *advanced users* of the Bernese GPS Software who wish to gain insight into *global applications of the GPS*. Let us start by mentioning a few basic facts:

- You will always estimate ERPs on top of an *a priori model*. These are characterized by a `*.ERP` file. If you want to estimate the ERPs *from scratch* you have to use an a priori model with all parameters set to zero.

- The (empirical) model for the ERPs is set up in the program GPSEST, it may be modified in program ADDNEQ, where you have to be aware of the fact that you may *simplify* but not generalize the model in ADDNEQ.
- If you solve for ERPs you will probably process exactly *one session* of one day in GPSEST and produce longer arcs using ADDNEQ. You should be aware of the fact that for the ERP estimates you may divide the time interval of the session into a number m of *subintervals* of equal length, and that within each subinterval each ERP (improvement on top of the a priori model) is represented by a polynomial.
- It should be pointed out that no model for diurnal and semidiurnal variations of the ERPs is applied in the Bernese GPS Software Version 4.0.
- $UT1 - UTC$ estimates are special in one respect (mentioned already above): due to the correlations with the orbital nodes mentioned above, it is *not* possible to solve for the *zero degree coefficient* of the polynomial pertaining to the first sub-interval of the session (if you introduce the orbital elements as unknowns in the same adjustment).

14.4.1 Options in GPSEST

Panel 4.5–2.4 handles many special requests in connection with parameter estimation. You have to enable here the parameter type `EARTH ROTATION PARAMETERS`. In this case the menu system gives you access to Panel 4.5–2.4.4, where you may first define the total number m of PARAMETER SETS (=total number of polynomials for each EOP). Specifying e.g. $m = 12$ means that you use 2-hour bins to model Earth rotation.

```
4.5-2.4.4 |            PARAMETER ESTIMATION: EARTH ROTATION PARAMETERS

      Model:
        TOTAL NUMBER OF PARAMETER SETS          > 3   <

      Earth Rotation (ERP) and Nutation Parameters (NUT):
                    # param./set (0-4)  default a priori sigma
        X-POLE           > 2 <              >        <        (mas)
        Y-POLE           > 2 <              >        <        (mas)
        UT1-UTC          > 2 <            >           <       (msec)
        DELTA EPSILON    > 0 <              >        <        (mas)
        DELTA PSI        > 0 <              >        <        (mas)

        CONTINUITY BETWEEN SETS            > ERP  <  (NO, ERP, NUT, BOTH)
        CONSTRAIN DRIFTS TO ZERO           > NO   <  (NO, ERP, NUT, BOTH)

      # of Values per Set Stored in Files:
        BERNESE POLE FILE      > 3  <       IERS POLE FILE      > 3  <
```

Subsequently you may select the number n of parameters for each of the five $EOPs$. The polynomial degree q is simply $q = n - 1$. $n = 0$ means that the corresponding parameters are *not* set up.

If you solve for polynomials of degree $q > 0$ you have to be aware of the fact that your estimates will in general *not* be continuous at the sub-interval boundaries. You may ask for continuity for the pole components x_p, y_p, and for $UT1-UTC$ by specifying `CONTINUITY BETWEEN SETS` "`ERP`"; if you want to enforce continuity for the two nutation parameters you specify "`NUT`", and you specify "`BOTH`" if you want to have continuity for all parameters. Attention: If you ask for continuity in the case $q = 0$ this actually means that you model the EOPs by *one* parameter for the entire session! If you ask for continuity in the case $q = 1$, you actually model the EOPs as polygons (standard procedure at CODE). You might ask for continuity of the first derivative, an option that only makes sense for a polynomial degree $q > 1$. We recommend to use $q = 1$ and to ask for continuity. Information concerning the other options is available in the help panel corresponding to Panel 4.5–2.4.

It was mentioned that it is not possible to solve for all orbit parameters *and for UT1-UTC*. Therefore the menu system allows you to introduce a priori weights for each parameter in each sub-interval. You have to constrain UT as suggested in the panel below in order to avoid singularities.

```
4.5-2.4.4   PARAMETER ESTIMATION: EARTH ROTATION PARAMETERS (XP, YP, DT)

       FROM                 TO                  SIG XP    SIG YP    SIG DT
  yy mm dd hh mm ss     yy mm dd hh mm ss       milli arc sec      milli sec
                                                *.****    *.****    .*******
> 96 06 09 23 59 59 < > 96 06 10 12 00 00 < >        <>        <> .0000001 <
> 96 06 10 12 00 00 < > 96 06 11 00 00 00 < >        <>        <>          <
> 96 06 11 00 00 00 < > 96 06 11 12 00 01 < >        <>        <>          <
```

If you defined the EOPs according to the above two panels, GPSEST will generate an output (in the general OUTPUT file) of the following type:

```
EARTH ROTATION PARAMETERS:
-------------------------

CRD. REQ.     (")        RMS        (")/DAY      RMS      (")/DAY**2
              (S) (DT)   RMS        (S)/DAY      RMS      (S)/DAY**2
---------------------------------------------------------------------

 X    1    0.0010080  0.0000900    -0.0007370  0.0001536
 Y    1    0.0017345  0.0001013    -0.0013315  0.0001621
 DT   1    0.0000000  0.0000000    -0.0002006  0.0000064
```

14.4.2 Options in ADDNEQ

Program ADDNEQ (Menu 4.8.1) allows the user to combine normal equation systems generated by GPSEST. ADDNEQ allows you to re-consider some aspects of EOP estimation. The panel generated by ADDNEQ (Panel 4.8.1–2.3 , see below) is similar to that generated by GPSEST, we only comment on some special features.

```
4.8.1-2.3 |      ADD NORMAL EQUTATION SYSTEMS: EARTH ROTATION PARAMETERS

      Earth Rotation (ERP) and Nutation Parameters (NUT):
                    # of parameters         a priori sigma
                     per set (0-4)   1st request    following requests
        X-POLE           > 2 <       > 0.      <       > 0.      <     (mas)
        Y-POLE           > 2 <       > 0.      <       > 0.      <     (mas)
        UT1-UTC          > 2 <       > 0.000001  <     > 0.        <   (msec)
        DELTA EPSILON    > 2 <       > 0.0001  <       > 0.      <     (mas)
        DELTA PSI        > 2 <       > 0.0001  <       > 0.      <     (mas)

        CONTINUITY BETWEEN SETS      > ERP  <           (NO, ERP, NUT, BOTH)
        CONSTRAIN DRIFTS TO ZERO     > ERP  <           (NO, ERP, NUT, BOTH)

        SPEC. OPTIONS (0, 2: CONTIN.BETWEEN SETS, 3:CODE H3)    > 0   <
        CONSTRAIN DAILY RETROGRADE X-Y FREQUENCY TO ZERO (0/1)  > 0   <

      # of Values per Set Stored in Files:
        BERNESE POLE FILE      > 3  <       IERS POLE FILE      > 3  <
```

Let us assume that ADDNEQ obtains a series of `*.NEQ` files pertaining to series of adjacent days. The resolution of the ERPs is given by the individual EOPs and will *formally not be altered by* ADDNEQ. You may ask for continuity of EOP estimations between subsequent days (`SPECIAL OPTIONS = 2`), you may ask that the EOPs are represented by *one polynomial of degree 1* over the entire time interval covered by all the files (`SPECIAL OPTIONS = 3`).

`CONSTRAINING RETROGRADE TERMS` is an option which is only of importance if you are interested in a sub-daily resolution of the EOPs. Please contact the Bernese team if you actually need this option!

There is one more important difference between the EOPs estimation in GPSEST and ADDNEQ: whereas in GPSEST the corrections to the a priori pole are modeled as polynomials, the *absolute values and UT1R* (obtained from $UT1$ by removing the tidal variations with periods $<$ 35 days) are represented as polynomials in ADDNEQ.

15. Initial Phase Ambiguities and Ambiguity Resolution

15.1 Motivation

The unknown number of cycles in the observation equations (9.17a) and (9.17b), the initial phase ambiguity parameters, have to be estimated in a first step as real-valued parameters. However, it is known, that these parameters actually have to be integer numbers. To *resolve* the ambiguities means (in our terminology) to assign the correct integer numbers to the real-valued estimates. Let us introduce the following notations:

$\mathbf{x}_1$	...	is the column array containing all non-ambiguity parameters,
$\mathbf{x}_1^0$	...	are the corresponding a priori values,
$\mathbf{x}_2$	...	is the column array containing all ambiguity parameters,
$\mathbf{x}_2^0$	...	are the corresponding a priori values, and
$\bar{\mathbf{x}}_2^0$	...	are the corresponding known true (integer) numbers. Moreover
Ψ	...	is the array containing the terms “computed” (model function),
$\mathbf{L}$	...	is the array containing all measurements (terms “observed”),
l, l'	...	are the arrays of reduced measurements (terms “observed – computed”),
$\mathbf{w}, \mathbf{w}'$	...	are the arrays containing the residuals.

Let us inspect the effect of introducing the known (integer valued) ambiguities into the normal equation system. In the case of the float solution we may write the observation equations in matrix form as

$$(\mathbf{A}_1\ \mathbf{A}_2) \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} - \underbrace{(\mathbf{L} - \Psi(\mathbf{x}_1^0, \mathbf{x}_2^0))}_{\mathit{l}} = \mathbf{w} \tag{15.1}$$

($\mathbf{A}_1$ and $\mathbf{A}_2$ are the parts of the first design matrix corresponding to the non-ambiguity resp. ambiguity parameters). The corresponding system of normal equations is

$$\begin{pmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} \\ \mathbf{N}_{21} & \mathbf{N}_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1^T \mathbf{P} \mathit{l} \\ \mathbf{A}_2^T \mathbf{P} \mathit{l} \end{pmatrix} = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix} \tag{15.2}$$

($\mathbf{P}$ is the weight matrix). Eliminating $\mathbf{x}_2$ from eqn. (15.2) we obtain

$$(\mathbf{N}_{11} - \mathbf{N}_{12}\mathbf{N}_{22}^{-1}\mathbf{N}_{21}) \cdot \mathbf{x}_1 = \mathbf{b}_1 - \mathbf{N}_{12}\mathbf{N}_{22}^{-1}\mathbf{b}_2\ . \tag{15.3}$$

Assuming that the ambiguity parameters are known we may write

$$\mathbf{A}_1\mathbf{x}_1 - \underbrace{(\mathbf{L} - \Psi(\mathbf{x}_1^0, \bar{\mathbf{x}}_2))}_{l'} = \mathbf{w}' \,, \tag{15.4}$$

which gives

$$\mathbf{N}_{11}\mathbf{x}_1 = \mathbf{A}_1^T\mathbf{P}l' = \mathbf{b}_1' \,. \tag{15.5}$$

We may write

$$l - l' = \Psi(\mathbf{x}_1^0, \bar{\mathbf{x}}_2) - \Psi(\mathbf{x}_1^0, \mathbf{x}_2^0) = \mathbf{A}_2 \cdot (\bar{\mathbf{x}}_2 - \mathbf{x}_2^0) = \mathbf{A}_2\mathbf{dx}_2 \tag{15.6}$$

and therefore

$$\mathbf{N}_{11}\mathbf{x}_1 = \mathbf{A}_1^T\mathbf{P}l - \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2\mathbf{dx}_2 \,. \tag{15.7}$$

This last equation shows how the normal equation system changes if the ambiguities have been resolved (fixed on their integer values). Fixing ambiguities considerably reduces the number of parameters and the solution will get much more stable. It should be pointed out, that usually the majority of unknown parameters actually are the ambiguities. How do the solutions improve if the ambiguities have been resolved? The answer depends strongly on the ratio between the number of unknown non-ambiguity parameters and the number of measurements which are used for the estimation of these parameters (the length of the observing sessions for static applications). Figure 15.1 shows the effect of ambiguity resolution if only the receiver coordinates and few troposphere parameters are estimated in a regional (European) network (for details see [*Mervart*, 1995]).

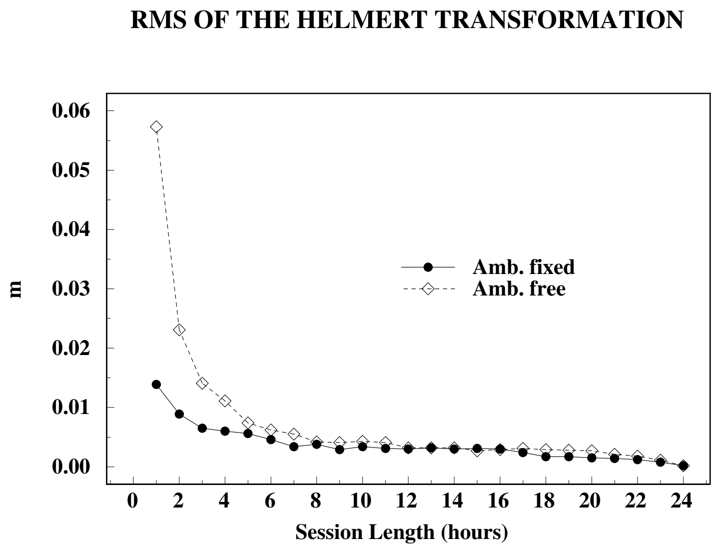
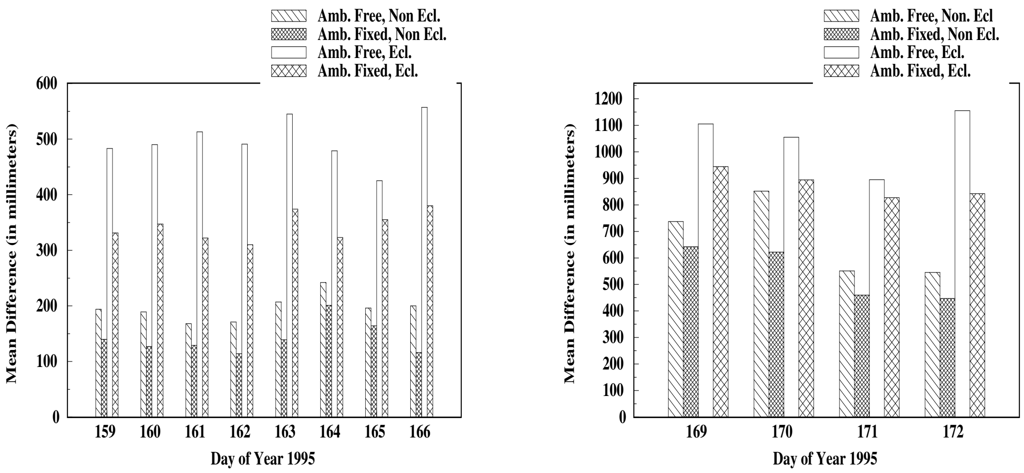


Figure 15.1: Rms of a 7-parameter Helmert Transformation with respect to the "True" Coordinate Set.

In this case the main effect may be seen for session length up to 4 hours. However, the second important advantage of the ambiguity fixed solutions is the significantly reduced number of parameters which have to be stored in the memory. This saves RAM and speeds up processing considerably. If many parameters are estimated (orbits, Earth orientation parameters etc.) ambiguity resolution improves also the results of much longer sessions (3-days sessions used in CODE for IGS processing). Figure 15.2 shows the improvement in the estimated orbits (for details see [*Mervart et al.*, 1995]):



(a) test 1

(b) test 2

Figure 15.2: Orbit Quality Estimated from Discontinuities at Day Boundaries (Eclipsing and Non-Eclipsing Satellites)

The ambiguity fixed solution is the official IGS CODE solution since June, 1995.

15.2 Theory

There are many methods how to resolve the ambiguities. Some of them are very sophisticated, some quite simple, but most of them consist of two steps:

Step 1: The ambiguities are estimated as real numbers together with other parameters.

Step 2: The integer values of the ambiguities are resolved using the results of Step 1 (the real-valued ambiguities and the variance-covariance matrix). Usually statistical tests are performed to resolve the ambiguities in a reliable way.

The Bernese GPS Software uses double difference observations and therefore the double difference ambiguities are estimated. Single difference (between receivers) ambiguities are then stored in the single difference header files. For each session and each baseline we have to select one single difference bias $n^j_{Fk\ell}$ as reference and actually our estimated ambiguity parameters are the differences

$$n^{ij}_{Fk\ell} = n^i_{Fk\ell} - n^j_{Fk\ell} \, . \tag{15.8}$$

Usually, the ambiguity with the maximum number of observations is selected as reference. If there are N single difference ambiguities for one session and one baseline, there are at most $N-1$ linearly independent unknown ambiguity parameters. If there is an epoch when all the single difference phase measurements were initialized again, the session breaks up into two parts and for each part one reference ambiguity *must* be selected. In that case only $N-2$ ambiguity parameters may to be estimated. The set of measurements corresponding to exactly one reference ambiguity is called an *observation cluster* (different from an ambiguity cluster, see below). In the following we will assume to have only one observation cluster.

Figure 15.3 shows the satellite visibility plot for a short (several minutes) session. For short sessions there is usually one (or more) satellite(s) which was(were) observed all the time. One of these satellites may be selected as reference satellite (and the corresponding ambiguity as reference ambiguity).

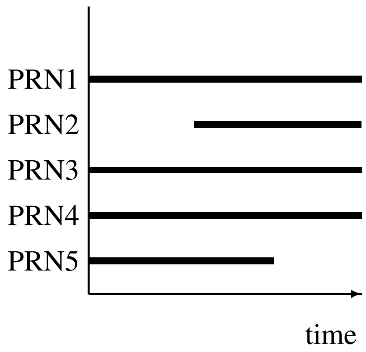


Figure 15.3: Satellite Visibility Plot for a Short Session and a Short Baseline

For longer sessions the situation is different:

- No satellite is observed during the entire session,
- there are periods during which only few satellites were observed, for very long baselines there may be even periods during which only one or two satellites were observed.

Typically, for long baselines and sessions we obtain a satellite visibility plot as in Figure 15.4:

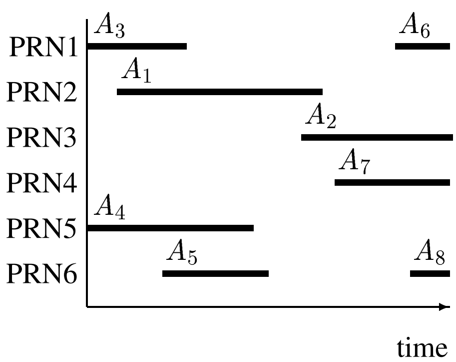


Figure 15.4: Satellite Visibility Plot for a Long Session and a Long Baseline

In the case of Figure 15.4 the program selects (single difference) ambiguity A_1 (maximum number of observations) as a reference. After the first ambiguity resolution step (real-valued ambiguities are estimated) a detailed inspection usually shows that the (double difference) ambiguities $A_2 - A_1, A_6 - A_1, A_7 - A_1, A_8 - A_1$ have large a posteriori rms errors. On the other hand, the parameters $A_3 - A_1, A_4 - A_1, A_5 - A_1$ have small rms errors. This result is a consequence of the selection of the reference ambiguity. If A_2 would have been selected as reference the parameters $A_6 - A_2, A_7 - A_2, A_8 - A_2$ would have small a posteriori rms errors and the parameters $A_1 - A_2, A_3 - A_2, A_4 - A_2, A_5 - A_2$ big ones. The following conclusions may be drawn:

- Depending on the selected reference certain differences between single difference ambiguities and the selected reference ambiguity are well established, other differences have large a posteriori rms errors.
- It is difficult to resolve *all* ambiguities if long sessions are processed because for each particular selection of a reference ambiguity some ambiguity parameters will have large a posteriori rms errors.

These considerations show that it is necessary to *optimize* the forming of (double) differences. Assuming that $n^j_{Fk\ell}$ denotes our reference ambiguity, we are therefore resolving either the double difference ambiguity parameter

$$n^{ij}_{Fk\ell} = n^i_{Fk\ell} - n^j_{Fk\ell} \tag{15.9}$$

directly or the difference between two of these terms

$$n^{i_1 i_2}_{Fk\ell} = n^{i_1 j}_{Fk\ell} - n^{i_2 j}_{Fk\ell} \, , \tag{15.10}$$

which, as a matter of fact, is a double difference ambiguity again. Every possible double difference ambiguity is covered by one of the two equations and any double difference ambiguity may be checked and possibly resolved. The resolved ambiguities are saved in the observation header files. We resolve the double difference ambiguities but for book-keeping reasons store single difference ambiguities in the files. It does not make sense to say that a single difference ambiguity is resolved without specifying the reference ambiguity. Therefore we introduce the term *ambiguity cluster*, which is the set of (single difference) ambiguities which are resolved relative *to each other*. In the single difference observation files the L_1, L_2 and L_5 ambiguities are stored. (This actually is redundant because the L_5 ambiguity is nothing else but the plain difference between the L_1 and L_2 ambiguities. The reason to store L_5 ambiguities, too has to be seen in the fact that L_5 ambiguities may sometimes be resolved a priori, see the ambiguity resolution strategies below.) Figure 15.5 shows the relevant part of a header file. Each ambiguity has its integer value (initialized to zero) and its cluster number. If two ambiguities refer to the same cluster number, this implies that they have been resolved relative to each other (see e.g. ambiguities 1, 4, and 5 in Figure 15.5). Any double difference ambiguity created from two such single difference ambiguities is then known. The ambiguities are set up by the programs SNGDIF and MAUPRP. Each satellite has at least one ambiguity.

```
AMB SAT EPOCH  WLF    L1-AMBIG.   CLUS    L2-AMBIG.   CLUS    L5-AMBIG.   CLUS
  1   1     1  1/1       681360.   69       530932.   69           0.     1
  2   1  2843  1/1           0.     2           0.     2           0.     2
  3   1  2879  1/1           0.     3           0.     3           0.     3
  4  31     1  1/1       633381.   69       493546.   69           0.     4
  5  31  1108  1/1      -145987.   69      -113753.   69           0.     5
  6  31  2758  1/1           0.     6           0.     6           0.     6
 ..
 ..
 ..
 ..
 63  27     1  1/1      1551327.   69      1208828.   69           0.    81
 67  16  1799  1/1           0.    67           0.    67           0.    86
 68  16  2182  1/1       279589.   69       227907.   69           0.    87
 69   2     1  1/1      1307750.   69      1019028.   69           0.    90
 70   7     1  1/1       709357.   69       552748.   69           0.    93
 71   5  1800  1/1      1334718.   69      1040042.   69           0.    97
 72   6     1  1/1      1518492.   69      1183242.   69           0.   100
```

Figure 15.5: Ambiguities Stored in Single Difference Phase Header File

15.3 Ambiguity Resolution Algorithms

There are four ambiguity resolution strategies implemented in the Bernese GPS Software Version 4.0: `ROUND`, `SIGMA`, `SEARCH` and `QIF` which may be selected in menu 4.5–1. (strategies `ELIMIN` and `NO` do not try to resolve any ambiguities). Different algorithms use different combinations of the L_1 and L_2 observations. This aspect will be discussed in Section 15.4. Let us mention, that for the QIF strategy both L_1 and L_2 observations are required, the SEARCH strategy gives the best results with observations on both frequencies, too, and that the remaining strategies ROUND and SIGMA work actually with one carrier (or one linear combination) only. It is possible to use 2 linear combinations in one run, but the ambiguity resolution is performed independently on both carriers.

15.3.1 NO Algorithm

If you specify "`YES`" for option `INTRODUCE WIDELANE` or `INTRODUCE L1 AND L2` in Panel 4.5–1, the ambiguities resolved in previous runs will be introduces as known values. All other ambiguities (or all ambiguities, if you specify "`NO`" for option `INTRODUCE`) will be introduced as unknown parameters and estimated. These ambiguities may or may not be pre-eliminated according to options in Panel 4.5–2.4.8.

15.3.2 ELIMI Algorithm

The only difference between `NO` and `ELIMI` is that if you select `ELIMI` strategy the ambiguities will be pre-eliminated before the normal equation matrix inversion. It is just the same as if you select `NO` strategy and "`BI`" option for ambiguity parameters in Panel 4.5–2.4.8.

15.3.3 ROUND Algorithm

This is the simplest ambiguity resolution strategy which only rounds the real-valued estimates to the nearest integers without using any variance-covariance information. Actually there is usually no need to use this strategy. The SIGMA strategy can do exactly the same work, if necessary. We do not recommend to use the strategy ROUND on baselines longer than a few kilometers. It is not a safe strategy.

15.3.4 Sigma-Dependent Algorithm

Let x_i, x_j be two (double difference) ambiguity parameters (relative to the same reference ambiguity). For each parameter x_i we compute the a posteriori rms error in the initial least-squares adjustment:

$$m_i = \sigma_0 \sqrt{Q_{ii}}\ , \tag{15.11}$$

where Q_{ii} is the corresponding element of the cofactor matrix. For the difference $x_i - x_j$ the a posteriori rms error is

$$m_{ij} = \sigma_0 \sqrt{Q_{ii} - 2 \cdot Q_{ij} + Q_{jj}}\ . \tag{15.12}$$

The rms errors m_i and m_{ij} of every possible double difference ambiguity (see eqn. (15.9) and (15.10)) are first sorted in ascending order of their rms errors. Within one iteration step the N_{max} best determined ambiguities (or differences between ambiguities) are then resolved (rounded to nearest integers), *provided*

- the corresponding a posteriori rms error m_i, m_{ij} is compatible with σ_0 ($m_i \leq \sigma_{max}$ or $m_{ij} \leq \sigma_{max}$), and
- that within the confidence interval $(x_i - \xi m_i, x_i + \xi m_i)$ or $(x_{ij} - \xi m_{ij}, x_{ij} + \xi m_{ij})$ there is exactly one integer number.

N_{max}, σ_{max} and ξ are input parameters of the program GPSEST (see Panel 4.5-1.1). In the next iteration step the integer values are introduced for the resolved ambiguities and for the resolved differences between ambiguities (see eqn. (15.7)). The iteration process is terminated, if:

1) all ambiguities have been resolved, or if

2) in the last step no ambiguity could be resolved based on the above criteria.

The iteration process described above may be applied to every linear combination. It may be used in the baseline mode, in the session mode, or even if several sessions are treated in the same program run. We recommend to use this strategy in two cases:

1) Only single-frequency measurements are processed, but the session is long (several hours). The baselines should not be to long (less than 20 km).

2) High quality code measurements are available on both frequencies. In this case it is possible to use the Melbourne-Wübbena linear combination and the corresponding strategy (see Section 15.4). The baselines may be very long (up to several thousand kilometers). The sessions have to be long too (several hours).

15.3.5 Search Algorithm

The SEARCH strategy is related to the so-called FARA (Fast Ambiguity Resolution Approach) algorithm (see [*Frei and Beutler*, 1990], [*Frei*, 1991]). The following information from the initial least-squares adjustment is used:

x $= (x_1, \ldots, x_u)^T$, the part of the solution vector consisting of all real-valued (double difference) ambiguities, where u is the number of double difference ambiguities,

Q, the corresponding cofactor matrix, and

σ_0^2, the a posteriori variance factor.

From the a posteriori variance factor and the corresponding cofactor matrix the standard deviation m_i for the ambiguity parameter x_i or the standard deviation m_{ij} for the difference x_{ij} between two ambiguity parameters $x_i,\ x_j$ may be computed:

$$m_i = \sigma_0 \sqrt{Q_{ii}}\ , \quad m_{ij} = \sigma_0 \sqrt{Q_{ii} - 2 \cdot Q_{ij} + Q_{jj}}\ . \tag{15.13}$$

Choosing a confidence level α and using Student's distribution we compute the upper and lower range-width ξ for the integer valued alternative parameter x_{Ai} or for the difference x_{Aij} between two such parameters. Thus

$$x_i - \xi \cdot m_i \leq \quad x_{Ai} \quad \leq x_i + \xi \cdot m_i\ , \quad i = 1, 2, \ldots, u \tag{15.14}$$

$$x_{ij} - \xi \cdot m_{ij} \leq \quad x_{Aij} \quad \leq x_{ij} + \xi \cdot m_{ij}\ , \quad i, j = 1, 2, \ldots, u\ , \quad i \neq j\ . \tag{15.15}$$

All possible combinations of integer values which meet the conditions (15.14) and (15.15) are used to form alternative ambiguity vectors

$$\mathbf{x}_{Ah},\ \ h = 1, \ldots, N$$

to the initial ambiguity estimate **x**. These alternatives are generated in forming all possible combinations of vector components using the integer values within corresponding confidence ranges. Each of these alternative vectors is introduced into a subsequent adjustment. The integer ambiguities are treated in these adjustments as known quantities. The resulting standard deviations

$$\sigma_h,\ \ h = 1, \ldots, N$$

are indicators for the success of the process: the integer vector $\mathbf{x}_h$ yielding the smallest standard deviation is selected as the final solution, unless

1) its standard deviation is not compatible with the standard deviation σ_0 of the ambiguity-free solution (the fraction σ_h/σ_0 is too high), or

2) there is another vector $\mathbf{x}_q$ yielding an almost identical standard deviation (fraction $\sigma_q/\sigma_h \approx 1$).

The maximum allowed fraction $(\sigma_h/\sigma_0)_{max}$ and the minimum discrimination fraction $(\sigma_q/\sigma_h)_{min}$ are input options in Panel 4.5–1.3. In order to reduce the computation time and decrease the number of alternative vectors one more condition is introduced if both frequencies (L_1 and L_2) are processed. Using the geometry-free linear combination (see Chapter 9) we may write

$$L^{ij}_{4k\ell} + I^{ij}_{k\ell}\left(1 - \frac{f_1^2}{f_2^2}\right) = \lambda_1 x^{ij}_{1k\ell} - \lambda_2 x^{ij}_{2k\ell} \; , \tag{15.16}$$

if the real-valued ambiguities $x^{ij}_{1k\ell}, x^{ij}_{2k\ell}$ are taken into account. Instead of these real-valued ambiguities we may use the alternative (integer) values $x^{ij}_{A1k\ell}, x^{ij}_{A2k\ell}$ to compute the right-hand site of eqn. (15.16):

$$\lambda_1 x^{ij}_{A1k\ell} - \lambda_2 x^{ij}_{A2k\ell}$$

The difference

$$\mid (\lambda_1 x^{ij}_{1k\ell} - \lambda_2 x^{ij}_{2k\ell}) - (\lambda_1 x^{ij}_{A1k\ell} - \lambda_2 x^{ij}_{A2k\ell}) \mid$$

is actually the difference between the ionosphere bias which was estimated during the initial ambiguity-free solution and the ionosphere bias which would be the result of the alternative ambiguity-fixed solution. The difference has to be very small (see option `SEARCH WIDTH FOR GEOMETRY-FREE LC` in Panel 4.5–1.3).

It is almost mandatory to use the SEARCH strategy in rapid static mode. If both frequencies are available (L_1 and L_2 measurements are processed) usually several minutes of data are sufficient to resolve the ambiguities and achieve an accuracy of about a centimeter. If only one frequency is processed the observation interval has to be longer (usually about 30 minutes of data are sufficient). In rapid static mode usually only short (up to several kilometers) baselines are processed.

The disadvantage of our SEARCH strategy has to be seen in the fact, that either all the ambiguities or none are resolved. This may cause problems if long sessions are processed and/or very long baselines are involved (see Figure 15.4).

15.3.6 QIF (Quasi Ionosphere-Free) Algorithm

We neglect the troposphere bias in eqns. (9.17a) and (9.17b) and do not explicitly write the receiver and satellite indices k, ℓ, i, j. Then the simplified form of the double difference observation equations reads as

$$L_1 \;=\; \varrho - I + \lambda_1\, n_1 \tag{15.17}$$

$$L_2 \;=\; \varrho - \frac{f_1^2}{f_2^2} \cdot I + \lambda_2\, n_2 \tag{15.18}$$

The corresponding equation for the ionosphere-free linear combination may thus be written as

$$L_3 = \varrho + B_3 = \varrho + \frac{c}{f_1^2 - f_2^2}\,(f_1\, n_1 - f_2\, n_2) \; . \tag{15.19}$$

The initial least-squares adjustment using both frequencies L_1 and L_2 gives real-valued ambiguity estimates b_1 and b_2 and we may compute the corresponding ionosphere-free bias $\tilde{B}_3$ as

$$\tilde{B}_3 = \frac{c}{f_1^2 - f_2^2}\,(f_1\, b_1 - f_2\, b_2) \; . \tag{15.20}$$

This bias may be expressed in narrow-lane cycles (one cycle corresponding to a wavelength of $\lambda_3 = c/(f_1 + f_2) \approx 11$ cm, see Chapter 9):

$$\begin{aligned} \tilde{b}_3 &= \frac{\tilde{B}_3}{\lambda_3} = \tilde{B}_3 \cdot \frac{f_1 + f_2}{c} = \frac{f_1}{f_1 - f_2}\, b_1 - \frac{f_2}{f_1 - f_2}\, b_2 \\ &= \beta_1\, b_1 + \beta_2\, b_2 \ . \end{aligned} \tag{15.21}$$

Denoting the correct (resolved) integer ambiguity values by n_{1i} and n_{2j} (i and j are not the satellite indices) and introducing the associated L_3-bias

$$b_{3ij} = \beta_1\, n_{i1} + \beta_2\, n_{2j} \tag{15.22}$$

we may use the difference between the real-valued and integer L_3-bias

$$d_{3ij} = |\tilde{b}_3 - b_{3ij}| \tag{15.23}$$

as a criterion for the selection of the "best" pair of integers n_{1i}, n_{2j}. However, many pairs n_{1i}, n_{2j} give differences d_{ij} of the same (small) order of magnitude. These pairs lie on a narrow band in the (n_1, n_2) space. The equation for the center line of this band is

$$\beta_1\, n_{i1} + \beta_2\, n_{2j} = \tilde{b}_3 \ . \tag{15.24}$$

The band-width is essentially given by the rms of the bias $\tilde{b}_3$. A unique solution only results if it is possible to limit the search range. The principle is shown in Figure 15.6.

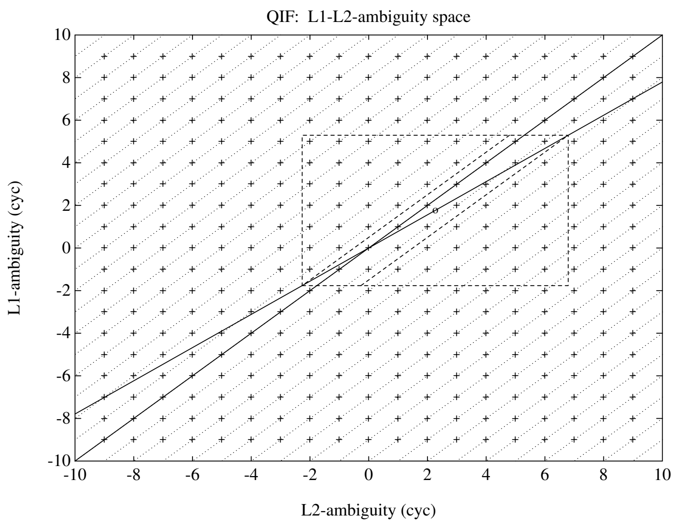


Figure 15.6: Search Ranges in (n_1, n_2) Space

The solid line corresponding to eqn. (15.24) goes through the real valued estimate (b_1, b_2) (shown as ₒ in figure) as well as through the point $(n_{1,i}, n_{2,j})$ which is accepted as "true" solution. This line represents an ionosphere–free combination (constant ionosphere-free bias). The second solid in Figure 15.6 represents the constant wide-lane ambiguity (accepted as "true" value) and goes through the point $(n_{1,i}, n_{2,j})$, too. The dashed rectangle represents a search range in (n_1, n_2) space and the dashed trapezoid represents the search range in (n_1, n_5) space – equation (15.29).

The Role of the Ionosphere

For baselines longer than about 10 km processing of the two frequencies L_1 and L_2 separately does *not* give sufficiently good initial real valued estimates b_1 and b_2 due to the influence of the ionospheric refraction. Two types of models to reduce the ionospheric biases are considered (see also Chapter 13):

1) Satellite and Epoch Specific Ionosphere Estimation: one ionospheric correction $I_k^i(t_j)$ for satellite i, receiver k and epoch (t_j) is estimated. Estimating these parameters without any a priori constraints would be equivalent to processing the ionosphere-free linear combination. If we want to resolve the integer ambiguities it is necessary to constrain these parameters to within a few decimeters. This constraining may be achieved by introducing an artificial observation

$$I_k^i(t_j) - I_{k,apr}^i(t_j) = 0 \tag{15.25}$$

for each epoch with a non-zero a priori weight. The actual values $I_{k,apr}^i(t_j)$ may stem from an ionosphere model, in many cases (baselines up to 500 km) even $I_{k,apr}^i(t_j) = 0$ may be sufficient. It is of course necessary to pre-eliminate all epoch-specific ionosphere parameters $I_{k,apr}^i(t_j),\ i = 1, 2, \ldots, n_s$ (n_s is the number of satellites per epoch) after having processed epoch t_j, because a "terrible" number of parameters would have to be handled in the normal equation system after n_e epochs.

2) Deterministic Model: single-layer models developing the electron content in a layer of infinitesimal thickness in a height of about 350 km above the surface of the Earth into a series of harmonical coefficients in latitude and hour angle of the Sun. Such a model should be used if long baselines (500 km – 2000 km) are processed.

A combination of the two types of models may be used.

Implementation of the QIF Strategy

Let us denote by b_{1i}, b_{1i_1}, b_{1i_2} the (real-valued) double difference L_1 ambiguities. Similarly by b_{2j}, b_{2j_1} and b_{2j_2} the corresponding L_2 ambiguities. Now, we check whether the pair

$$b_{1i}\ , \quad b_{2j}$$

or the pair

$$b_{1i_1} - b_{1i_2}\ , \quad b_{2j_1} - b_{2j_2}\ ,$$

which, as a matter of fact, is a pair of double difference ambiguities again, meets the requirements to be close to integers and may be accepted as the correct pair of integer ambiguities. Let us explain the procedure in more detail. We compute the rms error for each L_3 ambiguity bias $\tilde{b}_3$ associated with a pair b_{1i}, b_{2j} or with a pair of differences $b_{1i_1} - b_{1i_2}$, $b_{2j_1} - b_{2j_2}$:

$$\sigma = \sigma_0 \cdot \sqrt{\beta_1^2\, Q_{11} + 2\, \beta_1\, \beta_2\, Q_{12} + \beta_2^2\, Q_{22}}\ , \tag{15.26}$$

where

$$Q_{11} = Q(b_{1i}, b_{1i})\ , \quad Q_{12} = Q(b_{1i}, b_{2j})\ , \quad Q_{22} = Q(b_{2j}, b_{2j}) \tag{15.27}$$

in the case of pair b_{1i}, b_{2j} ($Q(\ldots)$ is an element of the cofactor matrix) or

$$\begin{aligned} Q_{11} &= Q(b_{1i_1}, b_{1i_2}) - 2\, Q(b_{1i_1}, b_{1i_2}) + Q(b_{1i_1}, b_{1i_2}) \\ Q_{12} &= Q(b_{1i_1}, b_{2j_1}) - Q(b_{1i_1}, b_{2j_2}) - Q(b_{1i_2}, b_{2j_1}) + Q(b_{1i_2}, b_{2j_2}) \\ Q_{22} &= Q(b_{2j_1}, b_{2j_1}) - 2\, Q(b_{2j_1}, b_{2j_2}) + Q(b_{2j_2}, b_{2j_2}) \end{aligned} \tag{15.28}$$

in the case of pair of differences $b_{1i_1} - b_{1i_2},\ b_{2j_1} - b_{2j_2}$. We sort the ambiguity pairs in ascending order of their σ-values. For the ambiguity pair (or pair of the differences) with the smallest σ (if this σ is lower than the specified $\sigma_{\max}$) we define the search ranges

$$\begin{gathered} \tilde{n}_1 = \mathrm{nint}(\mathrm{b}_1) \pm \mathrm{i}\ , \quad \mathrm{i} = 0; 1; \ldots; \mathrm{i}_{\max} \\ \tilde{n}_5 = \mathrm{nint}(\mathrm{b}_1 - \mathrm{b}_2) \pm \mathrm{k}\ , \quad \mathrm{k} = 0; 1; \ldots; \mathrm{k}_{\max} \\ \tilde{n}_2 = \tilde{n}_1 - \tilde{n}_5 \end{gathered} \tag{15.29}$$

and for each pair $\tilde{n}_1, \tilde{n}_2$ of integers within the search range we compute the test value (15.23)

$$d_3 = |\beta_1\ (b_1 - \tilde{n}_1) + \beta_2\ (b_2 - \tilde{n}_2)|\ . \tag{15.30}$$

The pair associated with the smallest value d_3 is accepted as a solution, unless

$$d_3 \geq d_{\max}\ , \tag{15.31}$$

where $d_{\max}$ is a user-defined maximum value. If no ambiguity set passed the test we proceed to the next pair of ambiguities associated with the second smallest σ. After having accepted one pair the entire least-squares adjustment and the procedure described above are repeated. The ambiguities are thus resolved iteratively. All or only a subset of ambiguity pairs may be resolved in the iteration process.

15.4 Resolution Strategies

Let us now give comments and recommendations concerning the selection of the appropriate ambiguity resolution strategy. All algorithms described in the previous section may be used either in the baseline mode or in the session mode. However, for the sake of efficiency, we recommend to resolve the ambiguities in the baseline mode (processing each baseline separately) and then introduce the resolved ambiguities as known quantities into the subsequent session processing. There are several aspects determining the optimal ambiguity resolution strategy. Before selecting the strategy the following questions should be answered:

1) Are phase measurements available on both carriers? If yes, are there high quality code measurements on both carriers, too? (This is often not true under the A/S regime.)

2) How long is the baseline?

3) How long is the session?

If only single frequency data are processed, there are not many possibilities how to resolve the ambiguities. Either the SEARCH algorithm (for short sessions) or the SIGMA algorithm (for long sessions) must be used. Only short (several kilometers) baselines should be processed.

If both frequencies are available, we have the following options:

1) For very short baselines (up to several kilometers) the ambiguities may be resolved independently on L_1 and L_2 using the SIGMA algorithm (it is possible to do that in one program run, if both carriers are processed). This case is similar to processing single frequency data.

2) For longer baselines (up to approximately 2000 km) it is possible to process both carriers together and to try to resolve both (L_1 and L_2) ambiguities in the same run. The recommended strategies are QIF for long sessions and baselines of almost arbitrary length and SEARCH for short sessions and short baselines.

3) Use of the so-called wide-laning technique. In this case each baseline is processed twice. First the L_5 or Melbourne-Wübbena linear combination is processed, the wide-lane (L_5) ambiguities are resolved and stored in the observation header file. In the subsequent run the L_3 linear combination is processed, the wide-lane ambiguities are introduced as known, and the narrow-lane (L_1) ambiguities are resolved.

It was stated above that in the first step of the third strategy (wide-laning technique) either the L_5 or the Melbourne-Wübbena linear combination is processed. Using L_5 (phase observations only) gets satisfactory results up to baseline lengths of about 100 km (or longer if a good a priori ionosphere model is used). In this case the processing steps are:

- Ambiguity-free L_3 solution saving the coordinate results to get good a priori coordinates for the next step.
- Wide-lane (L_5) ambiguity resolution fixing *all* coordinates on the values obtained in the previous step.
- L_3 solution introducing the resolved L_5 ambiguities and solving for the *narrow-lane* ambiguities. In this third step the estimation of station-specific troposphere parameters is highly recommended (see Chapter 12 because the geometrical term ρ (including the tropospheric refraction) has to be accurately known – considering the small wavelength. Note that the estimation of troposphere parameters is always recommended for long baselines when doing narrow-lane ambiguity resolution or when using the QIF strategy.

The approach given above (using phase observations only) does not seem to have advantages over the QIF strategy.

Resolving wide-lane ambiguities using the Melbourne-Wübbena linear combination is very reliable and almost baseline-length independent. However, the quality of the code measurement is crucial.

The recommended strategies are summarized in Table 15.1:

	Baseline Length		
	short < 10 km	middle 10–100 km	long 100–2000 km
short session	SEARCH	SEARCH	–
long session	QIF or SIGMA $L_1 \& L_2$	QIF	QIF or SIGMA Melb.-Wüebb.

Table 15.1: Ambiguity Resolution Strategies

16. Satellite and Receiver Clocks

In this chapter we describe how to obtain and use satellite and receiver clock information. Although the Bernese GPS Software uses double differences the clocks play an important role. The observations stemming from different receivers should all be measured simultaneously, synchronized with GPS time on the millisecond level because of the satellite clock dithering under SA. The receiver clocks should be synchronized on the microsecond level with respect to GPS time.

An application, where the clocks play an important role, is precise single point positioning using accurate satellite clock information. The IGS is providing, apart from accurate orbits, also accurate satellite clock solutions. These clock readings can e.g. be used to obtain coordinate estimates at the 50 cm level using only code observations of one single receiver.

16.1 Satellite Clocks

Satellite clock information is available either in the RINEX navigation data or in the precise orbit (SP3) file format. Both these formats have to be converted to a format suitable for the Bernese GPS Software. The RINEX navigation file has to be converted to the Bernese broadcast file format by the program RXNBV3, see Chapter 7 and Panel 2.7.2. This broadcast format contains both the satellite orbit and clock information. A separate clock file can be generated from the Bernese broadcast file using the SATCLK program, Panel 3.8.

The SP3 file format can be converted with the program PRETAB, Chapter 8. This program generates two separate files, one for the satellite orbit and one for the clock information. The orbits will be in the Bernese tabular format and the clocks in the Bernese satellite clock format.

For a description of file formats we refer to Chapter 23.

Satellite clocks are usually modeled by a low degree (1 or 2) polynomial. Therefore, when generating a clock file with program PRETAB the user is requested to enter, a title for the clock file, an interval length and a polynomial degree. We recommend to use a polynomial degree of 2 for each 12-hour interval.

16.2 Receiver Clocks

Receiver clock information is not readily available and consequently has to be estimated prior to forming single or double differences. In the Bernese GPS Software this is done by the program CODSPP, Chapter 10. This program performs a single point positioning using only code observations.

For this step satellite clocks are required! Because satellite clocks are mandatory in CODSPP, the program can read both file formats containing satellite clock information; the broadcast and the satellite clock file formats.

The single point positioning step allows the microsecond level synchronization of the receivers with respect to GPS time. This alignment to GPS time is achieved by using the satellite clock information. The estimated receiver clock corrections will be saved in both the code and phase zero-difference observation files.

Under SA (Selective Availability), mainly due to the clock dithering, the rms of the single point positioning will be of the order of 25 meters only. Without SA the rms should be around 5 meters. Even large rms values of up to 300 meters should cause no real problems later on in the processing, because 300 meters correspond to 1 microsecond. Nevertheless one should have a careful look at such results.

Within the IGS there are several Analysis Centers providing satellite clock estimates. These estimates are, however, in most cases not aligned to GPS time but rather to one particular reference clock. Only the clock estimates of CODE and the combined IGS clocks are aligned to GPS time. The clock offset differences between GPS time and a particular reference clock may be as large as 1 microsecond. This is something one should be aware of when using satellite clock information from different sources!

16.3 Precise Single Point Positioning

The IGS provides precise satellite clock information in addition to precise orbit information. The (IGS) clock precision is estimated to be better than 1 nanosecond (30 cm). This satellite clock information, together with the orbits, makes it possible to perform precise single point positioning. With the precision of both, the orbits and the clocks, being well below the noise of the code observations the positioning accuracy depends mainly on the code noise. For most geodetic receivers the noise of the code observations is around 50 cm under AS (anti-spoofing) conditions and some 20 cm without AS.

To make full use of the precise clock information we will have to repeat the PRETAB step, with special options, to generate a new satellite clock file which contains the actual values from the precise orbit file and not a polynomial fit. For this purpose the polynomial degree and interval should both be set to zero (“0”). The resulting satellite clock file will only contain satellite clock information every 15 minutes because this is the standard IGS sampling rate in the SP3 files. This clock file should only be used for precise point positioning!

After creating the satellite clock file (re)run the CODSPP program using this clock file and the standard orbit generated from the same precise orbit file as the clocks. **BE VERY CAREFUL** not to save the estimated receiver clocks in the phase zero-difference files as they would contain clock corrections only every 15 minutes. This would lead to severe problems when trying to process full-rate phase data.

16.4 Clock Input in Other Programs

It is possible to use the satellite clocks in programs MAUPRP and GPSEST but only in the satellite clock format. This possibility was created because in the early days of GPS not all receivers did measure near the full second. Therefore observation times could differ by hundreds of milliseconds. To be able to process double differences corresponding to different epochs one has to account for the satellite clock drift between "simultaneous" epochs. Of course this only works if SA is not turned on. Nowadays SA is permanently turned on but fortunately the receivers are all measuring within one millisecond of the full GPS second. So, although the possibility exists to use satellite clock information during MAUPRP and GPSEST processing, this is not really necessary.

Because the receiver clocks are written into the zero-difference observation files by the program CODSPP, this program has to be run. The receiver clock corrections are passed from the zero-difference files to the single-difference files when forming baselines with the program SNGDIF, Menu 3.4 and are then automatically used in all programs which process observations. For a description of the observation files and the clock corrections please refer to Chapter 23.

The Bernese GPS Software Version 4.0 does not include the possibility to estimate satellite clocks. Satellite clock estimation is currently under development and will be available in the next release of the software.

17. Antenna Phase Center Offsets and Variations

Motivation

In Chapter 9 we stated that GPS observations may be modeled by eqns. (9.14). Neglecting the clock corrections and atmospheric biases (which are not relevant for the topic of this chapter) we have an observation equation which (in the case of phase measurements) looks like

$$L^i_{Fk} = \varrho^i_k + \lambda_F \, n^i_{Fk} \; . \tag{17.1}$$

The term ϱ^i_k was declared to be "the geometrical distance between satellite i (at signal emission time) and receiver k (at signal reception time t)". However, if highest accuracy is required, this definition is not sufficient. We have to specify the exact points with respect to which the geometrical distance ϱ^i_k should be measured. These points are called "*antenna phase centers*".

17.1 Satellite Antenna Phase Centers

The precise position of the phase center of the satellite transmitting antenna with respect to the center of mass of the satellite is given in file `X:[GEN]SATELLIT.TTT`, which has to be specified in Panel 0.3.1 (see also Chapter 23):

```
SATELLITE SPECIFIC DATA                                                 08-MAR-95
-------------------------------------------------------------------------------
RADIATION PRESSURE MODEL : T950101    (ROCK MODEL T, FLIEGEL ET AL, 1992)

     BLOCK    ANTENNA OFFSETS (M)     MASS      DPO      P2     ROCK MODEL
PRN   NO.     DX       DY      DZ     (KG)    (1.E-8)  (1.E-9)   (T=1,S=2)

  1    3    0.2794  0.0000  1.0259    975.    -0.2132   0.5640      1    ok
  2    2    0.2794  0.0000  1.0259    878.2    0.0169   0.3178      1    ok
  3    3    0.2794  0.0000  1.0259    975.                          1    ok
  ...
```

It is assumed that the phase center is the same for the L_1 and L_2 carriers (and therefore also for all the linear combinations of L_1 and L_2) and that the phase center location remains constant. We will see, that the location of the *receiver antenna phase center* depends on the *direction* the signal is coming from. For satellite antenna we do im most cases not need to consider this direction-dependence because only the rays pointed towards the Earth (all of them having almost the same direction) are relevant. If the satellite passes through the Earth's shadow (for up to 55 minutes), however, it cannot orient itself correctly with respect to the Sun and the resulting mis-orientation of the satellite leads to biases in the geometrical distance (up to about 10 cm for very long baselines). There is also a dynamical effect caused by this mis-orientation: the solar-panels-axis is no longer perpendicular to the direction Sun$\longrightarrow$satellite and the solar radiation pressure force becomes very difficult to model. Both effects, the geometrical and the dynamical, are *not* taken into account by the software. The estimation of satellite antenna offsets (Panel 4.5–2.4.9) may be used to study the satellite antenna orientation in more detail.

To apply the antenna phase center positions from the file `X:[GEN]SATELLIT.TTT` correctly it is necessary to know, if the satellite positions (given in precise orbit file `*.PRE`) represent the positions of the center of mass (which is the normal case) or directly the positions of the antenna phase center. The user has to specify this in Panel 3.3–1 when running program ORBGEN. The Bernese and the IGS orbits always refer to the center of mass of the satellite.

17.2 Receiver Antenna Phase Centers and their Variations

In the case of receiver antennas the situation is more complicated because the GPS signals come from different directions (from different satellites) and the position of the antenna phase center depends on this direction. This direction-dependence is what we call *antenna phase center variations*. We also have to take into account that the antenna phase center positions and the antenna phase center variations are not identical for L_1 and L_2 carriers. Because the antenna phase center variations are antenna-type-dependent this effect has to be carefully modeled if different antenna types are used simultaneously. Mainly the relative station height is affected. The bias may reach values up to 10 cm – *independent* of the baseline length. If only antennas of the same type are used, the main effect is a scale factor in the network of up to about 0.015 ppm due to the fact that for long distances the antennas "see" the same satellite under different elevation angles.

In the Bernese GPS Software we use the following antenna phase center variation correction:

$$\Delta\phi(\alpha, z) = \Delta\phi'(\alpha, z) - \Delta\phi_0 + \mathbf{\Delta r} \cdot \mathbf{e} \, , \tag{17.2}$$

where

α, z ... is the azimuth and the zenith distance of the satellite,

$\Delta\phi_0$ ... is a constant phase offset uniquely defined by asking

$$\Delta\phi(\alpha, 0) = 0 \, ,$$

$\mathbf{\Delta r}$ ... defines the position of the "mean antenna phase center offset" with respect to the *antenna reference point*. This vector becomes uniquely defined by imposing the condition

$$\int\limits_{\alpha=0}^{2\pi} \int\limits_{z=0}^{z_{max}} \Delta\phi(\alpha, z) \; \sin z \, \mathrm{d}\, z \, \mathrm{d}\, \alpha = \text{min.} \quad (\text{e.g. } z_{max} = 75^{\circ}) \, .$$

The antenna reference points are defined for different antenna types in RINEX and IGS standards. For most antenna types the reference points are given in file `X:[GEN]ANTENNA.GRA`.

e ... denotes the unit vector in the direction antenna⟶satellite, and
$\Delta\phi'(\alpha, z)$ ... is the function modeling the phase center variations.

The correction $\Delta\phi(\alpha, z)$ is added to the geometrical distance ϱ_k^i of eqn. (17.1).

In the Bernese GPS Software two different model functions $\Delta\phi'(\alpha, z)$ may be used:

1) Piece-wise linear function in elevation (and optionally in the azimuth): polygon approach.

2) Spherical harmonic function of maximum degree n_{max} and maximum order $m_{max} \leq n_{max}$:

$$\Delta\phi'(\alpha, z) = \sum_{n=1}^{n_{max}} \sum_{m=0}^{n} \tilde{P}_{nm}(\cos 2z)\ (a_{nm}\ \cos m\alpha + b_{nm}\ \sin m\alpha)\ , \tag{17.3}$$

where $\tilde{P}_{nm}$ are normalized associated Legendre functions of degree n and order m, and a_{nm}, b_{nm} are the coefficients of the harmonic series development.

The receiver antenna phase center offsets $\mathbf{\Delta r}$ and the coefficient defining the function $\Delta\phi'(\alpha, z)$ are given in file `X:[GEN]PHAS_IGS.01`. The first part of this file looks like:

```
MODEL NAME: IGS_01    , PHASE CENTER VARIATIONS                      30-JUN_96
-------------------------------------------------------------------------------

RECEIVER TYPE          ANTENNA S/N  FREQ  PHASE CENTER OFFSETS (M)
ANTENNA  TYPE          FROM    TO    L*    NORTH    EAST     UP    FMT
********************  ****** ******   *   **.**** **.**** **.****   *
...
...
TRIMBLE 4000SSE            0 999999   1    0.0015 -0.0012  0.0751   2
TR GEOD L1/L2 GP                      2   -0.0011  0.0017  0.0692
...
```

In the first part of the file the phase center offset $\mathbf{\Delta r}$ is given (north, east, and up components). Please note the difference between the offsets for the first and the second carrier. In the example file above the antenna offsets are common for all the antennas of the same type (antenna numbers between 0 and 999999). The format flag (`FMT`) indicates that the antenna phase center variation model is given in the second part of the file:

```
...
RECEIVER TYPE        ANTENNA TYPE         FROM   TO      TYP   D(Z) D(A)
******************** ******************** ****** ******  ***    ***  ***
TRIMBLE 4000SSE      TR GEOD L1/L2 GP          0 999999    1      5  360

    A\Z      0      5     10     15     20     25     30     35     40  ...
L1   0   0.00   1.80   4.60   8.10  11.70  14.50  16.10  16.90  16.90  ...
L2   0   0.00   0.30   0.90   1.80   3.00   4.10   4.90   5.40   5.60  ...
...
```

In this case the model with piece-wise linear functions is used (`TYP=1`). The step in azimuth `D(A)` is 360°, which means that no azimuth dependence is considered. Let us point out that all phase center offsets and variations for different antenna types in file `X:[GEN]PHAS_IGS.01` are given *relative* to each other. This means, that one antenna type had to be selected as a reference by setting

$$\Delta\phi'(\alpha, z) - \Delta\phi_0 = 0 \; .$$

It may be seen in file `X:[GEN]PHAS_IGS.01` that the antenna phase center offsets and antenna phase center variations are different for both carriers. The antenna phase center corrections for a linear combination of L_1 and L_2 observations are computed by forming the correspondent linear combination of $\Delta\phi_{L1}(\alpha, z)$ and $\Delta\phi_{L2}(\alpha, z)$.

17.3 Estimation of the Receiver Antenna Phase Center Variations

The receiver antenna phase center offsets and variations may stem either from *chamber measurements* or from *estimations using GPS data*.

The chamber measurements are performed in anechoic test chambers where one specific antenna is mounted on a positioner that may rotate the antenna around two independent axes and shift it in three directions. The transmitting antenna is kept fixed while the receiving antenna (to be tested) is rotated through zenith angles from -90 to $+90$ degrees for various azimuth values. To rotate the test antenna as precisely as possible around the "mean" phase center for the actual measurements, the antenna is first shifted with respect to the center of rotation until the phase center variations with elevation are minimal and as symmetrical as possible for zenith angles corresponding to $-z$ and $+z$ degrees. Apart from the recording of the antenna phase values using a strip-chart recorder, the signal amplitude and axial ratio pattern are common measurements in chamber tests. These recordings have to be performed for both GPS carrier frequencies. Finally the location of the center of rotation with respect to a physical point on the test antenna, e.g. the *antenna reference points (ARP)* as defined by the IGS, has to be determined. For more details on chamber measurements we refer to [*Schupler et al.*, 1994].

Estimation of Phase Center Variations using GPS Data

The second possibility to determine the variations of the antenna phase centers is a GPS calibration campaign. The effort for setting up such a campaign is considerably smaller than collecting the chamber measurements. The resulting accuracy is comparable with that of the chamber tests but the disadvantage is that only *relative* corrections (differences between antenna types) may be estimated. Therefore at least one reference antenna has to be introduced as "known" (phase center offset and variations).

The Bernese GPS Software provides the possibility to estimate the antenna phase center offsets as well as the phase center variations. However, these options are not supported by the menu system. It is necessary to prepare the `I`-file manually using an ASCII editor (for details how to start programs without using the menu system we refer to Chapter 3). The relevant parts of the `I`-file is

```
ANTENNA PHASE CENTER PARAMETERS:
-------------------------------
                                                           *
PIECE-WISE LINEAR FUNCTIONS (CONTINUOUS)   =  1
COEFFICIENTS OF HARMONICAL FUNCTIONS       =  2     --> :  1

                                          ANT. NUMBERS     #PNTS/DEGR
       RECEIVER NAME    ANTENNA NAME       FROM    TO   FRQ. EL. AZ.  SIGMA (M)
F      **************** ****************  ****** ******  *   **  **   **.*****

--> :


RECEIVER ANTENNA OFFSETS:
------------------------                                  EST ?
                                                           *
N-COMPONENT OF ANTENNA                              --> :  0
E-COMPONENT OF ANTENNA                              --> :  0
U-COMPONENT OF ANTENNA                              --> :  0

                                          ANT. NUMBERS           SIGMA (M)
       RECEIVER NAME    ANTENNA NAME       FROM    TO   FRQ   HORIZ.    VERT.
F      **************** ****************  ****** ******  *  **.*****  **.*****

--> :
```

If piece-wise linear functions are used to model the phase center variations loose a priori constraints have to be put on each parameter to prevent the normal equation system from becoming singular because of the arbitrary constant $\Delta\phi_0$ in eqn. (17.2).

The representation of the antenna phase center variations by a series of spherical harmonics is physically more meaningful, in particular for the azimuth-dependent variations, but the polygon model may be introduced more easily into other software packages (linear interpolation between tabular values). You may even estimate spherical harmonics coefficients and save the results in the form of polygonal model (see internal names PHASRSG and PHASRSH in GPSEST N-file).

It is also clear from eqns. (17.2) that the station heights have to be fixed on known ground truth values. The remaining two components of the station position has to be fixed, too, if azimuth-dependent variations are determined. Fixing the horizontal position is not necessary if the antennas are rotated during the test campaign. The Bernese GPS Software handles this procedure by using the antenna-rotation file. An example for such a file may be found in X:[INX]EXAMPLE.AZI:

```
RECEIVER ANTENNA ORIENTATIONS                                           01-JAN-94
-------------------------------------------------------------------------------

RECEIVER TYPE          ANTENNA S/N   SESS  AZIMUTH
ANTENNA  TYPE          FROM    TO            DEG
********************  ****** ******  ****    ***

ROGUE SNR-8000          2104   2104  0011    270
DORNE MARGOLIN T

ROGUE SNR-8000          3104   3104  0011    270
DORNE MARGOLIN T
```

The user has to specify the name of the antenna-rotation file in N-file of GPSEST (the internal name

– see Chapter 3 – is `ANTAZI`). When estimating the phase center variations, one and the same set of coefficients may be estimated from several antennas of the same type (grouping of antennas).

We refer to [*Rothacher et al.*, 1995b] for details about the setup of the calibrations campaign. The first problem which has to be solved is how to determine the correct station positions (ground truth). It is e.g. possible to use the following approach: let us assume that the calibration campaign consists of two sessions (the sessions should be quite long - at least 24 hours) and let us further assume that only two antennas were present. We select one antenna as reference. We want to estimate the antenna phase center variations

$$\Delta\phi'(z)$$

(for the sake of simplicity we do not assume any azimuth dependence). Processing the first session (without estimating antenna phase center offsets and/or variations) the resulting relative position of both stations is corrupted by an unknown mean antenna phase center offset $\Delta\mathbf{r}$. If we exchange both antennas before the second session, the result of the second session is corrupted by $-\Delta\mathbf{r}$. The mean position (stemming from processing both sessions together) is correct. Now, it is possible to fix the station positions and to compute the coefficients defining the function $\Delta\phi'(z)$.

Let us include an example from a campaign described in [*Rothacher et al.*, 1995b]:

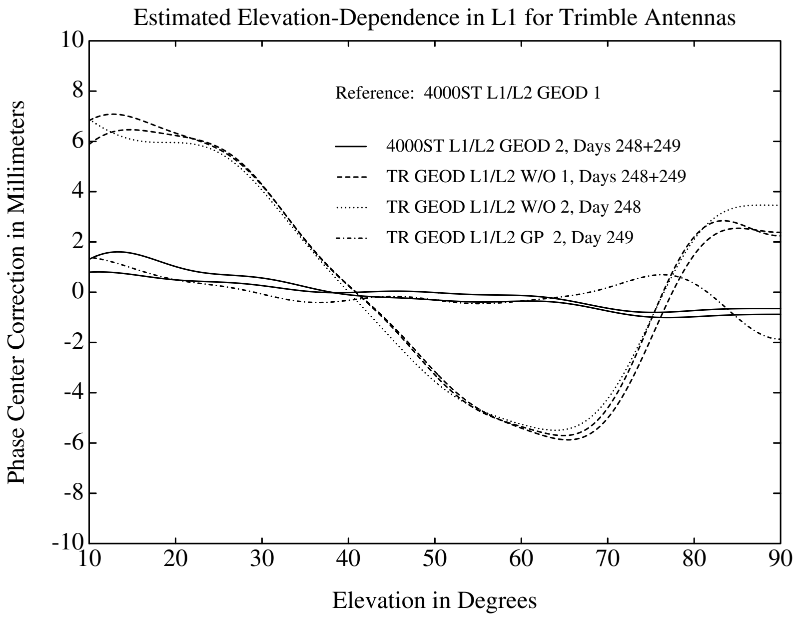


Figure 17.1: Trimble Antennas: Spherical Harmonics Development of Degree 10 Estimated from GPS Data

18. Combination of Solutions

18.1 Motivation

The increasing number of permanent GPS stations all over the world and the associated big number of observations to be processed ask for sequential processing methods. A "conventional" processing of all observations in one step using e.g. GPSEST may be appropriate for small campaigns (a few days with 24 hour sessions with about 10-20 sites). The computing power available today does no allow to go far beyond this limit.

The program ADDNEQ was therefore developed to compute multi-session solutions from the (statistically correct) combination of a set of single-session solutions. The theory of combining sequential solutions is well-known in geodesy since [*Helmert*, 1872]. Sequential adjustment techniques are in general independent of the observation types of the individual solutions. This implies e.g. that also results from different techniques (classical geodetic techniques or space techniques GPS, SLR, VLBI, DORIS) might be combined. Here we focus on the combination of GPS results, only.

Normal equations may be stored for a sequence of solutions including all possible types of unknown parameters (coordinates, troposphere, orbit parameters, earth rotation parameters, nutation parameters, center of mass, satellite antenna offsets, etc.).

The special features of the normal equation stacking methods, described in Section 18.3, allow an extremely rapid and flexible computation of many solution types, *without* going back to the original observations (e.g. new definition of the geodetic datum, specification of a priori sigmas for different parameters types, etc.).

We focus on applications and different processing strategies using normal equations in Section 18.4. The computation of velocities from campaign results or from results achieved from permanent GPS networks is one important applications. Another topic is the combination of GPS solutions of different Analysis Centers for the purpose of the *densification of the terrestrial reference frame* using GPS.

A description of the programs COMPAR and ADDNEQ is given in the Sections 18.6 and 18.7. Answers to "frequently asked questions" concerning parameter and normal equation handling conclude this chapter.

18.2 Basic Theory of Least-Squares Estimation

18.2.1 Least-Squares Estimation

The observation equations in the *Gauss-Markoff Model* (GMM) of full rank is given e.g. by [*Koch*, 1988]

$$\boldsymbol{E}(\boldsymbol{y}) = \boldsymbol{X}\boldsymbol{\beta} \quad ; \quad \boldsymbol{D}(\boldsymbol{y}) = \sigma^2 \boldsymbol{P}^{-1} \tag{18.1}$$

with

$\boldsymbol{X}$	$n \times u$ matrix of given coefficients with full rank $rank\ \boldsymbol{X} = u$; $\boldsymbol{X}$ is also called *design matrix*,
$\boldsymbol{\beta}$	$u \times 1$ vector of unknowns,
$\boldsymbol{y}$	$n \times 1$ vector of observations,
$\boldsymbol{P}$	$n \times n$ positive definite weight matrix,
n, u	number of observations, number of unknowns,
$\boldsymbol{E}(\cdot)$	operator of expectation,
$\boldsymbol{D}(\cdot)$	operator of dispersion,
σ^2	variance of unit weight (variance factor).

The observation equations of Chapter 9 may be written in this form. For $n > u$ the equation system $\boldsymbol{X}\boldsymbol{\beta} = \boldsymbol{y}$ is not consistent. With the addition of the residual vector $\boldsymbol{e}$ to the observation vector $\boldsymbol{y}$ one obtains a consistent but ambiguous system of equations, also called system of *observation equations*:

$$\boldsymbol{y} + \boldsymbol{e} = \boldsymbol{X}\boldsymbol{\beta} \quad \text{with} \quad \boldsymbol{E}(\boldsymbol{e}) = \boldsymbol{\emptyset} \quad \text{and} \quad \boldsymbol{D}(\boldsymbol{e}) = \boldsymbol{D}(\boldsymbol{y}) = \sigma^2 \boldsymbol{P}^{-1}. \tag{18.2}$$

Eqns. (18.1) and (18.2) are formally identical. $\boldsymbol{E}(\boldsymbol{e}) = \boldsymbol{\emptyset}$, because $\boldsymbol{E}(\boldsymbol{y}) = \boldsymbol{X}\boldsymbol{\beta}$, and $\boldsymbol{D}(\boldsymbol{e}) = \boldsymbol{D}(\boldsymbol{y})$ follows from the law of error propagation.
The *method of least-squares* asks for restrictions for the observation equations (18.1) or (18.2). The parameter estimates $\boldsymbol{\beta}$ should minimize the quadratic form

$$\Omega(\boldsymbol{\beta}) = \frac{1}{\sigma^2}(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})'\boldsymbol{P}(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \tag{18.3}$$

where $(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})'$ is the transposed matrix of $(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})$. The introduction of the condition $\Omega(\boldsymbol{\beta}) \rightarrow$ min. is necessary to lead us from the ambiguous observation equations (18.1) or (18.2) to an unambiguous normal equation system (NEQ system) for the determination of $\boldsymbol{\beta}$.
The establishment of minimum values for $\Omega(\boldsymbol{\beta})$ leads to a system of u equations $d\Omega(\boldsymbol{\beta})/d\boldsymbol{\beta} = \boldsymbol{\emptyset}$, also called *normal equations*.
The following formulae summarize the *Least-Squares Estimation* (LSE) in the Gauss-Markoff Model:
Normal equations:

$$\boldsymbol{X}'\boldsymbol{P}\boldsymbol{X}\widehat{\boldsymbol{\beta}} = \boldsymbol{X}'\boldsymbol{P}\boldsymbol{y} \tag{18.4}$$

Estimates:

$$\text{of } \boldsymbol{\beta}: \quad \widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{P}\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{P}\boldsymbol{y} \tag{18.5}$$

$$\text{of the (variance-)covariance matrix: } \boldsymbol{D}(\widehat{\boldsymbol{\beta}}) = \widehat{\sigma}^2(\boldsymbol{X}'\boldsymbol{P}\boldsymbol{X})^{-1} \tag{18.6}$$

$$\text{of the observations: } \widehat{\boldsymbol{y}} = \boldsymbol{X}\widehat{\boldsymbol{\beta}} \tag{18.7}$$

$$\text{of the residuals: } \widehat{\boldsymbol{e}} = \widehat{\boldsymbol{y}} - \boldsymbol{y} \tag{18.8}$$

$$\text{of the quadratic form: } \Omega \overset{1.}{=} \widehat{\boldsymbol{e}}'\boldsymbol{P}\widehat{\boldsymbol{e}} \overset{2.}{=} \boldsymbol{y}'\boldsymbol{P}\boldsymbol{y} - \boldsymbol{y}'\boldsymbol{P}\boldsymbol{X}\widehat{\boldsymbol{\beta}} \tag{18.9}$$

$$\text{of the variance of unit weight (variance factor): } \widehat{\sigma}^2 = \Omega/(n-u) \tag{18.10}$$

Degree of freedom / Redundancy:

$$f = n - u \tag{18.11}$$

Normal equation matrices:

$$\boldsymbol{X}'\boldsymbol{P}\boldsymbol{X}, \quad \boldsymbol{X}'\boldsymbol{P}\boldsymbol{y}, \quad (\boldsymbol{y}'\boldsymbol{P}\boldsymbol{y}) \tag{18.12}$$

This algorithm is used in the parameter estimation program GPSEST (Menu 4.5).

18.2.2 Parameter Pre-elimination

Pre-elimination of parameters is a basic procedure to reduce the dimension of the NEQ system *without* loosing information (apart from the parameters pre-eliminated). Here we do not give the mathematical proof. For more information see e.g. [*Brockmann*, 1996].
The pre-elimination formulas basically compute the effect of the pre-eliminated parameters on the other (remaining) parameters of the normal equation system. As a result the normal equation matrices (18.12) are modified. Pre-elimination therefore is NOT equivalent to cancelling the corresponding lines and columns of the normal equations.
Pre-elimination of parameters using covariance matrices as opposed to pre-elimination using normal equations is much easier. The determination of partial covariance matrices is identical to removing the corresponding rows and columns of the parameters which have to be eliminated from the covariance matrix.
Pre-elimination of parameters is possible with both, program GPSEST (Panel 4.5–2.4, option `PARAMETER PRE-ELIMINATION`) and program ADDNEQ (Panel 4.8.1–2, option `PARAMETER PRE-ELIMINATION`). It is the responsibility of the user to decide at which stage of the processing to pre-eliminate parameters from the NEQ system (mainly a question of processing time and disk space). More information is given in Section 18.4 and in the examples in Chapter 4.
Let us distinguish between the pre-elimination options `BI` (before Inversion), `AI` (after inversion), and `EP` (epoch-wise, GPSEST only):

`BI` : Pre-elimination of a parameter *before inversion*.
Used mainly for ambiguity parameters in GPSEST (if there remain unresolved ambiguities from previous GPSEST runs) or for troposphere parameters in ADDNEQ (if they were stored previously in the normal equations) to reduce the number of unknowns in the combined solution.

`AI` : Pre-elimination of a parameter *after inversion*.
Used in GPSEST and ADDNEQ to store only the parameters of interest in the normal equation files. Ambiguity parameters should be pre-eliminated using option `AI` in GPSEST if normal equations are stored.

EP : Pre-elimination of a parameter directly after each observation epoch.
Used in GPSEST for epoch-specific parameters as e.g. kinematic coordinates or stochastic ionosphere parameters.

18.2.3 Sequential Least-Squares Estimation

In this section we review the concept of sequential least-squares estimation techniques. The result of a LSE using all observations in one step is the same as when splitting up the LSE in different parts and combining the results later.
To prove the identity of both methods we first solve for the parameters according to the common adjustment in one adjustment step. Thereafter we verify that the same result is obtained using a sequential adjustment.
Let us start with the observation equations:

$$\begin{aligned} \boldsymbol{y}_1 + \mathbf{e}_1 &= \boldsymbol{X}_1\, \boldsymbol{\beta_c} \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{y}_1) = \sigma_1^2 \boldsymbol{P}_1^{-1} \\ \boldsymbol{y}_2 + \mathbf{e}_2 &= \boldsymbol{X}_2\, \boldsymbol{\beta_c} \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{y}_2) = \sigma_2^2 \boldsymbol{P}_2^{-1}. \end{aligned} \tag{18.13}$$

In this case we divide the observation array $\boldsymbol{y}_c$ (containing all observations) into two independent observation series $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$. We would like to estimate the parameters $\boldsymbol{\beta_c}$ common to both parts using both observation series $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$. We assume furthermore, that there are no parameters which are relevant for one of the individual observation series, only. This assumption is meaningful if we pre-eliminate "uninteresting" parameters according to Section 18.2.2.
The proof of the equivalence of both methods is based on the assumption that both observation series are independent.
The division into two parts is general enough. If both methods are leading to the same result we might derive formulae for additional sub-divisions by assuming one observation series to be already the result of an accumulation of different observation series.

18.2.3.1 Common Adjustment

In matrix notation we may write the observation equations (18.13) in the form:

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta_c} \end{bmatrix} \\ \text{with} \quad &\boldsymbol{D}(\begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix}) = \sigma_c^2 \begin{bmatrix} \boldsymbol{P}_1^{-1} & \emptyset \\ \emptyset & \boldsymbol{P}_2^{-1} \end{bmatrix} \end{aligned} \tag{18.14}$$

which is equivalent to

$$\boldsymbol{y}_c + \boldsymbol{e}_c = \boldsymbol{X}_c \boldsymbol{\beta_c} \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{y}_c) = \sigma_c^2 \boldsymbol{P}_c^{-1}. \tag{18.15}$$

The matrices $\boldsymbol{y}_c$, $\boldsymbol{e}_c$, $\boldsymbol{X}_c$, $\boldsymbol{\beta_c}$, and $\boldsymbol{P}_c^{-1}$ may be obtained from the comparison of eqn. (18.15) with eqn. (18.14). The independence of both observation series is given by the special form of the dispersion matrix (zero values for the off diagonal elements). Substitution of the appropriate values for $\boldsymbol{y}_c$, $\boldsymbol{X}_c$ and $\boldsymbol{\beta_c}$ in eqn. (18.4) leads to the normal equation system of the LSE:

$$\begin{bmatrix} \boldsymbol{X}_1' \boldsymbol{P}_1 \boldsymbol{X}_1 + \boldsymbol{X}_2' \boldsymbol{P}_2 \boldsymbol{X}_2 \end{bmatrix} \begin{bmatrix} \widehat{\boldsymbol{\beta}}_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1' \boldsymbol{P}_1 \boldsymbol{y}_1 + \boldsymbol{X}_2' \boldsymbol{P}_2 \boldsymbol{y}_2 \end{bmatrix}. \tag{18.16}$$

18.2.3.2 Sequential Least-Squares Adjustment

In a first step the sequential LSE treats each observation series independently. An estimation is performed for the unknown parameters using only the observations of a particular observation series. In a second step the contribution of each sequential parameter estimation to the common estimation is computed.
Starting with the same observation equations as in the previous section, eqns. (18.13), we may write

$$\begin{aligned} \boldsymbol{y}_1 + \boldsymbol{e}_1 &= \boldsymbol{X}_1\,\boldsymbol{\beta_1} \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{y}_1) = \sigma_1^2 \boldsymbol{P}_1^{-1} \\ \boldsymbol{y}_2 + \boldsymbol{e}_2 &= \boldsymbol{X}_2\,\boldsymbol{\beta_2} \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{y}_2) = \sigma_2^2 \boldsymbol{P}_2^{-1} \end{aligned} \tag{18.17}$$

or, in more general notation:

$$\boldsymbol{y}_i + \boldsymbol{e}_i = \boldsymbol{X}_i\,\boldsymbol{\beta_i} \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{y}_i) = \sigma_i^2 \boldsymbol{P}_i^{-1},\ i = 1,2 \tag{18.18}$$

where the vector $\boldsymbol{\beta_i}$ denotes the values of the common parameter vector $\boldsymbol{\beta_c}$ satisfying observation series $\boldsymbol{y}_i$ only.

<u>First step: Solving for each individual NEQ</u>

The normal equations for the observation equation systems $i = 1, 2$ may be written according to eqn. (18.4) as

$$\left[\ \boldsymbol{X}_i'\boldsymbol{P}_i\boldsymbol{X}_i\ \right]\left[\ \widehat{\boldsymbol{\beta}}_i\ \right] = \left[\ \boldsymbol{X}_i'\boldsymbol{P}_i\boldsymbol{y}_i\ \right] \tag{18.19}$$

$$\begin{aligned} \boldsymbol{D}(\widehat{\boldsymbol{\beta}}_i) &= \widehat{\sigma}_i^2 (\boldsymbol{X}_i'\boldsymbol{P}_i\boldsymbol{X}_i)^{-1} \\ &= \widehat{\sigma}_i^2 \boldsymbol{\Sigma}_i \quad \text{with} \quad i = 1,2. \end{aligned} \tag{18.20}$$

<u>Step 2: A posteriori LSE</u>

In this a posteriori LSE step the estimation for $\widehat{\boldsymbol{\beta}}_c$ is derived using the results of the individual solutions (18.19) and (18.20) obtained in the first step.
The pseudo-observation equations set up in this second step have the following form

$$\boldsymbol{y}_{II} + \boldsymbol{e}_{II} = \boldsymbol{X}_{II}\widehat{\boldsymbol{\beta}}_c \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{y}_{II}) = \sigma_c^2 \boldsymbol{P}_{II}^{-1} \tag{18.21}$$

or more explicitly:

$$\begin{bmatrix} \widehat{\boldsymbol{\beta}}_1 \\ \widehat{\boldsymbol{\beta}}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_{1_{II}} \\ \boldsymbol{e}_{2_{II}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{I} \end{bmatrix} \widehat{\boldsymbol{\beta}}_c \text{ with } \boldsymbol{D}(\begin{bmatrix} \widehat{\boldsymbol{\beta}}_1 \\ \widehat{\boldsymbol{\beta}}_2 \end{bmatrix}) = \sigma_c^2 \begin{bmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\emptyset} \\ \boldsymbol{\emptyset} & \boldsymbol{\Sigma}_2 \end{bmatrix}.$$

The results of the individual estimations $\widehat{\boldsymbol{\beta}}_i$ and $\boldsymbol{\Sigma}_i$ are thus used to form the combined LSE. The interpretation of this pseudo-observation equation system is easy: Each estimation is introduced as a new observation using the associated covariance matrix as the corresponding weight matrix.
The normal equation system may be written as:

$$\boldsymbol{X'}_{II}\boldsymbol{P}_{II}\boldsymbol{X}_{II}\widehat{\boldsymbol{\beta}}_c = \boldsymbol{X'}_{II}\boldsymbol{P}_{II}\boldsymbol{y}_{II} \tag{18.22}$$

or more explicitly

$$\begin{aligned}&\left[\begin{array}{c}\boldsymbol{I}',\boldsymbol{I}'\end{array}\right]\left[\begin{array}{cc}\boldsymbol{\Sigma}_1^{-1} & \emptyset \\ \emptyset & \boldsymbol{\Sigma}_2^{-1}\end{array}\right]\left[\begin{array}{c}\boldsymbol{I}\\ \boldsymbol{I}\end{array}\right]\widehat{\boldsymbol{\beta}}_c \\ &= [\boldsymbol{I}',\boldsymbol{I}']\left[\begin{array}{cc}\boldsymbol{\Sigma}_1^{-1} & \emptyset \\ \emptyset & \boldsymbol{\Sigma}_2^{-1}\end{array}\right]\left[\begin{array}{c}\widehat{\boldsymbol{\beta}}_1\\ \widehat{\boldsymbol{\beta}}_2\end{array}\right].\end{aligned} \tag{18.23}$$

Substituting the results for $\boldsymbol{\Sigma}_i^{-1}$ we obtain

$$\left[\begin{array}{c}\boldsymbol{X}_1'\boldsymbol{P}_1\boldsymbol{X}_1+\boldsymbol{X}_2'\boldsymbol{P}_2\boldsymbol{X}_2\end{array}\right]\ \widehat{\boldsymbol{\beta}}_c\ =\left[\begin{array}{c}\boldsymbol{X}_1'\boldsymbol{P}_1\boldsymbol{y}_1+\boldsymbol{X}_2'\boldsymbol{P}_2\boldsymbol{y}_2\end{array}\right] \tag{18.24}$$

which is identical with eqn. (18.19). This simple superposition of normal equations, also called *stacking* of normal equations, is always possible if the individual observation series are independent (which is the case if the dispersion matrix has the form (18.14)).

18.2.3.3 Computation of the Combined RMS

In the previous section we only considered the combined parameter estimation. Sequential LSE leads to identical results for the a posteriori estimate of the variance of unit weight:

$$\Omega_c = \sum_{i=1}^{m}\boldsymbol{y}_i'\boldsymbol{P}_i\boldsymbol{y}_i-\sum_{i=1}^{m}\boldsymbol{y}_i'\boldsymbol{P}_i\boldsymbol{X}_i\widehat{\boldsymbol{\beta}}_c \tag{18.25}$$

$$\widehat{\sigma}_c^2 = \left(\sum_{i=1}^{m}\boldsymbol{y}_i'\boldsymbol{P}_i\boldsymbol{y}_i-\sum_{i=1}^{m}\boldsymbol{y}_i'\boldsymbol{P}_i\boldsymbol{X}_i\widehat{\boldsymbol{\beta}}_c\right)/f_c. \tag{18.26}$$

The importance of the “third normal equation part” $\boldsymbol{y}'\boldsymbol{P}\boldsymbol{y}$ (see eqn. 18.12) is clearly seen in this formula. We refer to [*Brockmann*, 1996] for a complete discussion.

18.3 Special Features of Combining Normal Equations

Special features are “the salt in the soup” when dealing with normal equations. Here we present only some important ones. Most of them may be derived from general parameter transformation rules applied to normal equations (see [*Brockmann*, 1996]).

18.3.1 Constraining Parameters

In general, the observations of a given type are *not* sensitive to *all* parameters in a theoretical model. In this case the normal equations (NEQs) are singular.
Additional information, or *constraints*, must be introduced into the least-squares solution to make the normal equations non-singular. But also for parameters which would be estimated with a very high rms additional constraints may be useful. Let us introduce “exterior” information concerning the parameters

$$\boldsymbol{H}\boldsymbol{\beta}=\boldsymbol{w}+\boldsymbol{e}_w \quad \text{with} \quad \boldsymbol{D}(\boldsymbol{w})=\sigma^2\boldsymbol{P}_w^{-1} \tag{18.27}$$

where

$\boldsymbol{H}$ $r \times u$ matrix with given coefficients with $rank\ \boldsymbol{H} = r$,

r number of constraining equations with $r < u$,

$\boldsymbol{\beta}$ vector of unknown parameters with dimension $u \times 1$,

$\boldsymbol{w}$ $r \times 1$ vector of known constants,

$\boldsymbol{e}_w$ $r \times 1$ residual vector, and

$\boldsymbol{P}_w^{-1}$ dispersion matrix of the introduced constraining equations with dimension $r \times r$.

If the constraints are non-linear a linearization has to be performed through a first order Taylor series expansion.
We may interpret the constraints (18.27) as additional pseudo-observations or as *fictitious observations*. That leads us to the observation equations:

$$\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{w} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_y \\ \boldsymbol{e}_w \end{bmatrix} = \begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{H} \end{bmatrix} \widehat{\boldsymbol{\beta}} \text{ with } \boldsymbol{D}(\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{w} \end{bmatrix}) = \sigma^2 \begin{bmatrix} \boldsymbol{P}^{-1} & \boldsymbol{\emptyset} \\ \boldsymbol{\emptyset} & \boldsymbol{P}_w^{-1} \end{bmatrix} \tag{18.28}$$

or to the associated NEQ system:

$$(\boldsymbol{X'PX} + \boldsymbol{H'P}_w\boldsymbol{H})\widehat{\boldsymbol{\beta}} = \boldsymbol{X'Py} + \boldsymbol{H'P}_w\boldsymbol{w}. \tag{18.29}$$

The equation shows, that we may superpose the terms $\boldsymbol{H'P}_w\boldsymbol{H}$ and $\boldsymbol{H'P}_w\boldsymbol{w}$ to the original normal equation system to incorporate a priori information on the parameters. The values of these quantities have to be stored in the normal equation files (or in the SINEX files (see Section 23.8.12)). The terms have to be removed if a "free" solution (without any a priori constraints) has to be created.
Constraints may be introduced in GPSEST and ADDNEQ for the following parameter types:

- coordinates: absolute constraints (station weights), station fixing, free network constraints,
- velocities: absolute and relative (concerning sites) constraints,
- troposphere: absolute and relative (in time) constraints,
- orbit: Keplerian, dynamical, stochastic parameters,
- center of mass,
- earth rotation parameters: absolute constraints (UT1 and nutation absolute value has to be constrained to a VLBI value) and continuity constraints,
- satellite antenna offsets.

The most important features dealing mainly with the first three types of parameters are explained in more detail in the application part of this chapter.

18.3.2 Introducing Additional Parameters

The introduction of additional parameters is possible even if these parameter types have not been set up in the individual normal equations. Site motion parameters, or site velocities, are an example for such parameter types. It is necessary, that the influence of these parameters may be neglected within the individual normal equations. E.g. for the site velocities the influence of the site motion may actually be neglected for the time span of one day. How to estimate velocities is shown later on.
Another example for the set up of additional parameters is the estimation of Helmert parameters between individual solutions (see Section 23.8.13).

18.3.3 Independence of the A Priori Information

The a priori values of the parameters used is also stored in the normal equations. That is important because the normal equations refer to the parameter increments (the difference between the estimated final parameter values and the a priori values). The normal equations may be transformed to an arbitrary set of a priori parameter values if higher order terms of the original non-linear observation equations may be neglected. The transformation to new a priori values is done automatically (if necessary) without user interaction.

18.3.4 Free Network Constraints

Free network solutions are optimal to define the geodetic datum with a minimum number of constraints, without fixing or constraining particular site coordinates. This option is well-suited to analyse inconsistencies in the reference site coordinates.
A geodetic datum may be defined in the following way:

- An a priori network is defined by selecting a list of sites (in the Menu system of ADDNEQ "fixed" sites are used for this purpose).
- Helmert parameters may be specified (translations, rotations, scale). Depending on the selection of these parameters, the final parameter estimation has the property that the network results show no translations / rotations / scale with respect to the a priori network (this network has to be specified, too).

More information may be found in Section 18.7.3.

18.3.5 Reduction of the Number of Unknown Parameters

As opposed to adding new parameters it is also possible to reduce the number of unknown parameters in the normal equations. An important application is the reduction of the number of troposphere parameters. If you have e.g. estimated (and stored into NEQ files) 12 troposphere parameters per site and day, you have the possibility to reduce the number of parameters to 1, 2, 3, 4, or 6 values per day and site. This option is available in Panel 4.8.1–2.2, option `NUMBER OF PARAMETERS PER DAY`. See also Section 18.7.6.

18.3.6 Limitations of NEQ Stacking

Let us also mention what is *not* possible using ADDNEQ:

- model modifications which are highly time-dependent (e.g. a different tropospheric mapping function, different a priori tide model, etc.),
- ambiguity resolution,
- different basic observation types (e.g. to switch from L_1 and L_2 to L_3, etc.).

18.4 Applications and Strategies using Normal Equations

It is possible to combine results based on normal equations without loss of information. Figure 18.1 shows an example how results are combined at CODE. Starting from cluster solutions network solutions are created. These results are then used for long-arc applications. Finally weekly, monthly, or annual solutions may be created.

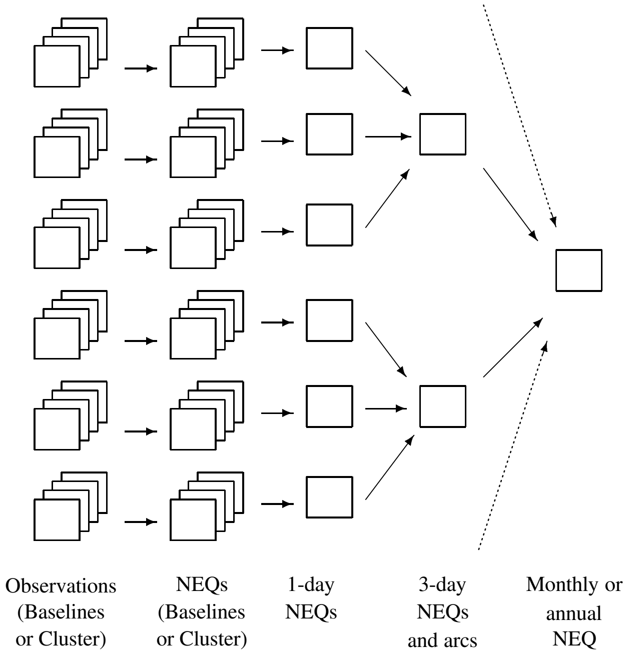


Figure 18.1: Combination of the Normal Equations of Different Processing Steps.

There is a wide area of applications for combination methods. Below, we briefly review some important applications when processing GPS observations.

Baseline processing mode:

Figure 18.1 demonstrates how baseline or cluster results are combined into a network solution.

The baseline processing scheme (also implemented in the BPE processing example; see also Chapter 4) has the advantage, that the computational burden *increases only linearly with the number of sites*. It is a disadvantage, however, that *inter-baseline correlations* are not taken into account. For highest accuracy requirements we therefore recommend to process all observations with GPSEST using the correct handling of the correlations (Panel 4.5–2, option `CORRELATIONS`, select `CORRECT`). For big networks this may not be possible due to limited computer resources (memory and computing time). As a compromise between statistical correctness and computational efficiency you may define *clusters* of observations and process each cluster using the option “correct correlations”. Afterwards you may combine the cluster normal equations (instead of baseline normal equations) into a network solution. The use of cluster definition files for processing clusters of observations is explained in Sections 23.8.26 and 23.8.27.

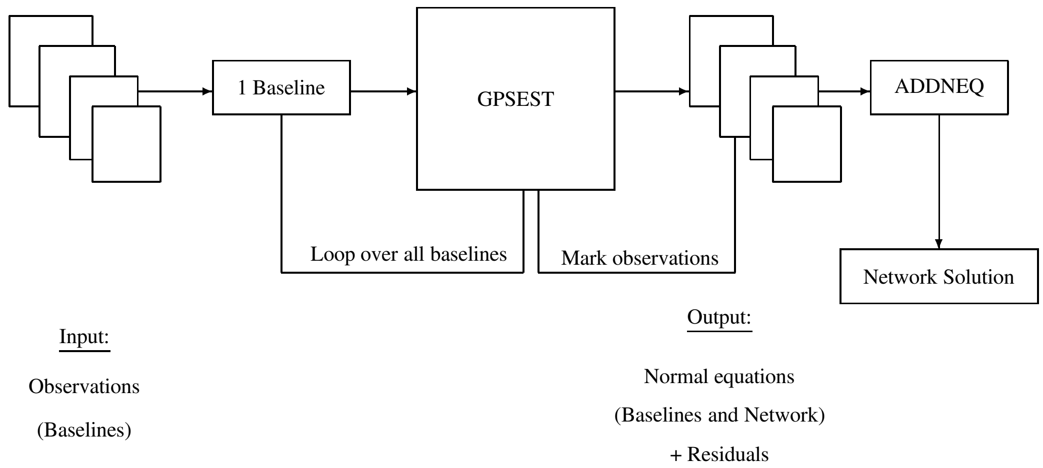


Figure 18.2: Processing Scheme based on Baseline (or Cluster) Processing.

Multi-Days Solutions:

The creation of weekly or monthly solutions from daily solutions is sometimes useful to reduce the variations in the coordinate solutions. The noise of e.g. weekly coordinate residuals is smaller by a factor of $1/\sqrt{7}$ in a weekly solution when compared to the daily solution.

Multi-Years Solutions or Multi-Campaigns Solutions:

The computation of "final" coordinates as a result of many days of continuous observations or several campaigns is the main goal of the combination of solutions. The program ADDNEQ was originally developed for this purpose. That includes also the detection of movements (estimation of velocities), which is described in Section 18.7.5.

Orbit Combination:

Orbit combination is probably not of great interest for the majority of users. The orbit combination method described in [*Beutler et al.*, 1996] and [*Brockmann*, 1996], (see also Chapter 8) is an extremely flexible tool for orbit determination purposes. Long arcs (e.g. 3-days-arcs) may be computed from short arcs (e.g. 1-day-arcs) in a very efficient way (gain of more than a factor of 10 in processing time). Many more options are available such as setting up stochastic parameters at the arc boundaries, splitting up of arcs, etc..

Combination of Solutions using Results of Different Processing Centers:

The combination of (GPS-) solutions derived by different Analysis Centers is a major activity within the IGS with the goal of densifying the International Terrestrial Reference Frame (ITRF). The *distributed processing concept* makes it possible that regional or local Analysis Centers (in the IGS naming convention: Regional Network Associated Analysis Centers RNAACs) may compute their sites of interest together with global IGS sites (also called *anchor sites*). These solutions may then be combined by Global Network Associated Analysis Centers (GNAACs) together with the global solutions of the IGS Analysis Centers to form a consistent network solution. It is not necessary in this concept that all

contributing sites are processed by one Analysis Center. Combination strategies and results are shown by e.g. [*Davies and Blewitt*, 1995] and [*Brockmann and Gurtner*, 1996].
The SINEX (see Section 23.8.12) exchange format is used for the combination of the results of different Analysis Centers (see Sections 7.3 and 23.8.12). Using the program SNXNEQ (see Section 7.3.3) it is possible to convert SINEX files (`.SNX`) into normal equation files (`.NEQ`), which may be used as input files for ADDNEQ. The combination of results derived from different software packages asks for the determination of normal equation rescaling factors to ensure that each contributing solution gets the "correct" weight. Rescaling factors may be specified using a special weighting (`.WGT`) file (see Section 23.8.13). The estimation of these factors using the methods of the *variance-covariance component estimation* is not supported, yet.

18.5 The Combination Programs ADDNEQ and COMPAR

Two programs are available to combine solutions in the Bernese GPS Software Version 4.0 : ADDNEQ (Menu 4.8.1) and COMPAR (Menu 5.4.1). ADDNEQ is based on normal equations and is able to handle all types of unknown parameters, COMPAR is based on covariance information of coordinates, only.
In general it is equivalent to combine solutions based on normal equations or based on covariance information. ADDNEQ is much more flexible, COMPAR is much simpler to use. The selection of the tool depends on the user requirements. We discuss both program.

18.6 Combination Program COMPAR

Menu 5.4.1 is used to prepare a run of program COMPAR. All important features of this program are activated using Panel 5.4.1.

```
5.4.1                        SERVICES: COORD. COMPARISON

  CAMPAIGN           >          <    (blank for selection list)

Input Files:
  COORDINATES        > SELECTED <    (blank for selection list)
  COVARIANCES        > NO       <    (NO, SAME, blank for selection list)
  A PRIORI COORD     > NO       <    (NO, blank for selection list)
  BASEL. DEFINITIONS > NO       <    (NO, blank for selection list)

  Use Plot Skeleton    > NO  <       (YES or NO) Name: U:\INP\COMPARP.INP

Output Files:
  COORDINATES        > NO       <    (NO, if not to be created)
  COVARIANCES        > NO       <    (NO, if not to be created)
  PLOT FILE          > NO       <    (NO, if not to be created)
  WEEKLY SUMMARY     > NO       <    (NO, if not to be created)
```

The program COMPAR is used to compare different coordinate sets (select input files `COORDINATES`) without allowing for additional Helmert parameters between the different sets. We mentioned already that the program is also suited to compare the coordinates using the associated variance-covariance information (select `COVARIANCES`). Keep in mind, that with this program you do not have the flexibility to change the constraints specified in GPSEST, or to change the geodetic datum. It is e.g. not possible to combine coordinate sets which are computed using different fixed or heavily constrained sites. We therefore recommend to use the program ADDNEQ if a statistically correct combination should be performed.

The program is well-suited to study coordinate repeatabilities and baseline results. By setting two options you may activate the printing of baseline repeatabilities in the output file. The first option is BASEL. DEFINITIONS in Panel 5.4.1 (see Section 23.8.25) and the second one has to be set in Panel 5.4.1–1, (Repeatability option; select LOCAL or GEOcentric). All this information may also be obtained using ADDNEQ. For an output description we refer to Section 18.7.7, because most of the output information is very similar to the output of ADDNEQ.

It is possible to create a summary file (WEEKLY SUMMARY) using COMPAR. Agencies participating in the IGS densification project may use this possibility to automatically create a summary file for their weekly submission of SINEX results.

18.7 Combination Program ADDNEQ

18.7.1 General Introduction

Most of the input options in ADDNEQ are identical to the options available in program GPSEST. This includes the handling of different parameter types (e.g. coordinates, troposphere, orbits, center of mass, etc.). We therefore will not repeat all different input options in detail, here. Furthermore, we refer to the available HELP panels if questions concerning a specific input option arise.
We put the emphasis on the differences with respect to the parameter estimation using GPSEST and to the additional features, which are available in ADDNEQ, only.

18.7.2 Differences to GPSEST

Below we summarize important differences between GPSEST and ADDNEQ:

- An a priori coordinate file has to be specified in GPSEST, only. ADDNEQ does not need such a file because the information is already stored in the normal equations. A coordinate file, specified in Panel 4.8.1, (see Figure 18.4) option UPDATE CRD. is only used as a master file to create a coordinate output file (specified in Panel 4.8.1–0, option COORDINATES).
- Fixing site coordinates in GPSEST is equivalent to not setting up these parameters as unknowns (not recommended, if normal equations are stored; see Section 18.8). Fixing site coordinates in ADDNEQ is equivalent to specifying an a priori sigma of 0.001 mm.
- Troposphere handling is much simpler than in GPSEST. In ADDNEQ you only have the possibility to specify a general absolute and a relative a priori sigma. These values are valid for all troposphere parameters stored in the normal equations. It is not possible to handle different sites in a different way (e.g. to constrain the troposphere parameters of some sites more than others).
- ADDNEQ features, which are NOT available in GPSEST:

- Velocity estimation (more details are given below).
- Creation of a SINEX file (see e.g. Section 23.8.12 and Section 7.3)
- Free network solutions (more details are given below).
- Long-arc computation based on one-day-arcs (not explained in detail in this documentation).
- Special handling of the earth rotation parameters (see Chapter 14).

18.7.3 Free Coordinate Solutions

Let us focus on some important aspects when generating so-called "free network solutions" (compare also Section 18.3.4):

```
4.8.1-1              ADD NORMAL EQUATION SYSTEMS: INPUT 1

     TITLE    > FREE NETWORK SOLUTION                                <

   Coordinates:
     FIXED STATIONS      >                  <   (blank: sel.list, ALL, NONE,
                                                 SPECIAL_FILE, $FIRST, $LAST)
     A PRIORI SIGMAS     > NO   <               (YES, NO)
     FREE SOLUTION COND. > YES <                (YES, NO)

   Velocities:
     FIXED STATIONS      > NONE             <   (blank: sel.list, ALL, NONE,
                                                 SPECIAL_FILE, $FIRST, $LAST)
     A PRIORI SIGMAS     > NO   <               (YES, NO)
     FREE SOLUTION COND. > NO  <                (YES, NO)
     INTRODUCE VELOC.    > NO  <                (YES, NO)
```

Figure 18.3: Panel 4.8.1–1 Options to Define the Geodetic Datum of a Solution.

- You may activate the free network solutions in Panel 4.8.1–1 (see Figure 18.3).
- "Fixed" stations have to be specified to define the a priori sites used for defining the reference network. Please select `FIXED STATIONS` using `blank`, using `ALL` sites, or using a `SPECIAL_FILE`. Do **not** use `A PRIORI SIGMAS` in this case.
- Identical free network options may also be defined for the velocities.
- You have to introduce Helmert constraints in Panel 4.8.1–1.1 (or Panel 4.8.1–1.2 for the velocities). Introduction of e.g. 3 translation constraints is comparable (in view of the number of constraints) with keeping one site fixed (one site velocity vector fixed, respectively). We recommend to introduce 3 translation constraints for the definition of the geodetic datum of the coordinates. If you would like to align your solution to a specific network (and not to ITRF given by the used IGS orbits) you may also specify 3 rotation constraints. The scale of your solution should be estimated from the GPS data without any restrictions from the a priori network.
  In the case of velocities we recommend to introduce 3 translation parameters. Scale and rotation constraints have the same effects as the translation constraints, if you specify `NO` "`FIX ON SPEC. VELOC.`" file in Panel 4.8.1 (see Figure 18.4).

- The residuals of each individual solution with respect to the combined solution (see Section 18.7.7) are computed after applying a 7-parameter transformation (independently of the specified Helmert parameters) using all sites (independently of the selected “fixed” sites). The application of the free network option has many advantages, in particular if you compare different solutions with only a small number of common sites or if an unique definition of the geodetic datum for **all** contributing solutions is difficult to realize.
- If a special file for fixing coordinates is specified (in Panel 4.8.1 (see Figure 18.4) for coordinates using the option `FIX ON SPEC. COORD.` or for velocities using the option `FIX ON SPEC. VELOC.`), the free network constraints are computed using the coordinate or velocity values of the selected “fixed” sites in these files instead of using the a priori coordinates originally used in GPSEST or instead of using a zero-velocity field). More information will be given in the next section.

```
4.8.1                         ADD NORMAL EQUATION SYSTEMS

  CAMPAIGN          >          <    (blank for selection list)

 Job Identification:
  JOB CHARACTER        >   <        (blank, or characters A - Z, 0 - 9)

 Input Files:
  NORMAL EQUATIONS      > SELECTED <    (blank:  sel.list)
  UPDATE CRD.           > NO       <    (NO: not used, blank: sel.list)
  FIX ON SPEC. COORD.   > NO       <    (NO: not used, blank: sel.list)
  A PRIORI VELOC.       > NO       <    (NO: not used, blank: sel.list)
  FIX ON SPEC. VELOC.   > NO       <    (NO: not used, blank: sel.list)
  PLATE TABLE NUVEL1    > NO       <    (NO: not used, blank: sel.list)
  COV. COMPONENT INTRO  > NO       <    (NO: not used, blank: sel.list)
  PRE-DEFINED BASELINES > NO       <    (NO: not used, blank: sel.list)
  SITES FOR REPEATABIL. > NO       <    (NO: not used, blank: sel.list)
```

Figure 18.4: The First Option Input (Panel 4.8.1) of ADDNEQ.

18.7.4 Fixing Coordinates or Velocities on Special Values

Fixing coordinates or velocities on given values (different from the original a priori values) is useful when defining the geodetic datum with fixed sites.

The option may be useful if data were processed using “bad” a priori coordinates (but not exceeding a few decimeters). For the final parameter estimation you then may define the geodetic datum by fixing site coordinates to more meaningful values (e.g. the ITRF92, ITRF93, or ITRF94 values).
The procedure works only if the differences (new − old a priori coordinates) are still in the linearity domain of the observation equations.

Constraining and also fixing of coordinates or velocities *without* specifying a special fixing file means that the constraints are set up with respect to the a priori values originally used in GPSEST when saving the normal equations. For coordinates the a priori values are taken from the normal equation file, in which a specific site appears first.
For velocities the default reference is a zero-velocity field.

If you specify a special fixing file (in Panel 4.8.1 (see Figure 18.4) for coordinates using option `FIX ON SPEC. COORD.` or for velocities using option `FIX ON SPEC. VELOC.`), the same restrictions are set up as without specifying these files, but now with $\boldsymbol{w} \neq \emptyset$ (see eqn. 18.27), with $\boldsymbol{w}$ as the difference between the original a priori values and the new specified a priori values. These new a priori values are used from the specified fixing file for the selected fixed sites, only.

Note that it is not possible to save normal equations in this case (see Section 18.8). Note also that *special fixing files are* **not** *used for the constraints* (set up using options `A PRIORI SIGMAS` in Panel 4.8.1–1 (see Figure 18.3)).

18.7.5 Site Velocity Estimation

The estimation of site velocities is an important application of ADDNEQ. Velocity estimation is possible, if you have processed data covering a long time span. The quality of individual coordinate estimates is an important factor as well (see e.g. [*Brockmann*, 1996]). It is easy to invoke a velocity estimation: Specify `YES` in Panel 4.8.1–1 (see Figure 18.3), option `A PRIORI SIGMAS`. Then you will get a list of the sites for which you may specify a priori sigmas in units of mm per year. This list is similar to the list you get if you specify `A PRIORI SIGMAS` for site coordinates. Without specifying a priori sigmas no site velocities are estimated. We recommend to use a value of e.g. 999.99 mm/yr per component if you would like to perform a free velocity estimation. We recommend to solve for horizontal velocities, only (to specify e.g. 0.01 mm/yr for the vertical components of the velocities) if you do not have very long time spans of data.
Velocities are also set up if you select "fixed stations" using Panel 4.8.1–1 (see Figure 18.3), option `FIXED STATIONS` for velocities. The consequences of "fixed" velocities was explained already in Section 18.7.3 and Section 18.7.4.
Estimating *one velocity common to several site occupations* is not supported by the menu system. If you edit the I-file of ADDNEQ (see Chapter 3 or Section 23.9) and place an asterisk "`*`" behind the *station number* (see Section 23.8.1) and the associated a priori velocity sigmas, identical velocities will be estimated for all sites with the same station number. If sites with different station numbers are processed you may use the station problem file (see Section 23.4.12) to change station numbers for this purpose. Keep in mind that in all other cases the site velocities are set up using the *station name*, only.

18.7.6 Tuning Troposphere Estimates

Troposphere parameterization very much depends on the size of the GPS network an the session lengths involved. It is not possible to come up with a list stating for all possible applications the number of troposphere parameters per day and site necessary and their associated a priori sigmas. You have to find the optimal parameterization and the optimal values for the a priori sigmas by tests of your own ("try and error"). In Chapters 4 and 12 you find more information.
ADDNEQ may assist you to find the optimum:

- Store the troposphere parameter information in the NEQ files in GPSEST (see Section 18.2.2).
- Use ADDNEQ and modify the troposphere options in Panel 4.8.1–2.2 (see Figure 18.5): Reduce the number of unknown parameters according to Section 18.3.5 using option `NUMBER OF`

PARAMETERS PER DAY and modifying the a priori sigmas ABSOLUTE and RELATIVE. Furthermore, it may be useful to force CONTINUITY BETWEEN the troposphere estimates of two consecutive NEQS (the specified relative sigmas are used for this purpose; pre-elimination of the troposphere parameters (option BI) is not allowed in this case).

- Check if the coordinate repeatabilities, listed in the ADDNEQ program output (see Section 18.7.7), are getting better or not.

```
4.8.1-2.2        ADD NORMAL EQUATION SYSTEMS: SITE-SPECIFIC TROPOSPHERE

    A priori Sigma:
       ABSOLUTE                          > 0.10  <     (meters)
       RELATIVE                          > 5.00  <     (meters)

    Modelling:
       CONTINUITY BETWEEN NEQS           > YES <      (YES, NO)
       NUMBER OF PARAMETERS PER DAY      > 000 <      (0: AS IN NEQ)
```

Figure 18.5: The Option Input (Panel 4.8.1–2.2) of ADDNEQ to modify the Parameterization and the a priori Constraints of the Troposphere.

18.7.7 Output Description

The ADDNEQ output file consists of the following parts:

LIST OF INPUT AND OUTPUT FILENAMES
: A list of the input files used and the stored output files (information given in the N-file).

LIST OF NORMAL EQUATION FILES
: A list of the normal equation files used (information given in the F-file) including basic information like the number of parameters contained in the normal equations, title, and the re-scaling value (may be specified using a .WGT file according to Section 23.8.13).

LIST OF STATIONS
: The list contains station number, station name (see Section 23.8.1), a flag if velocities are estimated (not in the example below), the total number of coordinate “observations”, and a table showing in which files each site was observed. The flags used in the table are explained at the bottom of the list.

```
TOTAL NUMBER OF STATIONS:    38
NUM  STATION       VELO R #FIL 12345
-----------------------------------------------------------------------------
186  CAGL 12725M003          2 XX
155  MATE 12734M008B         3 XX  X
122  MASP 31303M002          2 XX
176  VILL 13406M001          2 XX
154  MADR 13407S012          2 NN
158  ZIMM 14001M004          4 NNNN
...  .... .........          . .....
FLAGS:    W: WEIGHTS , F: FIXED, N: FREE NETWORK RESTRICTIONS, X: FREE ESTIM.
```

`INPUT OPTIONS`

A summary of important input options is included:

- a priori sigmas for coordinates and velocities,
- free network constraints,
- orbit model information,
- troposphere information.

`STATISTICS FOR ESTIMATED PARAMETERS`

The statistics of estimated parameters may look as follows (extracted from a solution covering 3 years of continuous global GPS data processing):

```
-----------------------------------------------------------------------------------
STATISTIC OF SOLVED FOR PARAMETERS     #PARAMETERS   #PRE-ELIMINATED        #NO-OBS
-----------------------------------------------------------------------------------

STATION COORDINATES                        309          36  (BEFORE INV)        0
SATELLITE ANTENNA OFFSETS                    6           0                     47
CENTER OF MASS                               3           0                      0
STATION VELOCITIES                         285           0                      0
-----------------------------------------------------------------------------------
NUMBER OF SOLVE FOR PARAMETERS             603          36                     47
-----------------------------------------------------------------------------------
```

For each parameter type the following statistical information is included: number of unknown parameters, number of pre-eliminated parameters (pre-elimination of particular sites from particular solutions may be achieved according to Section 23.4.12), and number of parameters with no observations (e.g. parameters that were set up originally with a priori sigmas, but did not figure in any observation equation).

`SHORT SOLUTION STATISTICS`

For the same example the solution statistics has the form:

```
TOTAL NUMBER OF PARAMETERS                    :    3148110
NUMBER OF OBSERVATIONS                        :  101705205
NUMBER OF SINGLE DIFF. FILES                  :      83491
-----------------------------------------------
SIGMA OF SINGLE DIFFERENCE OBSERVATION:    0.0038
SIGMA OF COORDINATE GROUP             :    0.0000
```

The solution summary contains information concerning the total number of parameters (including all unknowns which may have been pre-eliminated in previous runs), the number of observations, and the total number of processed single difference files. The value `SIGMA OF SINGLE DIFFERENCE OBSERVATION` ($\widehat{\sigma}^2$ in eqn. (18.6), same meaning as in program output of GPSEST) is an important indicator for the quality of the used receivers and for the quality of the solution (see Chapter 4). This value is furthermore used as the scaling factor for the estimated covariance matrix.

The value `SIGMA OF COORDINATE GROUP` is computed if only coordinates are involved in the combination. The value is derived as a group rms from weighted coordinate repeatabilities [*Brockmann*, 1996] and is better suited (gives a more realistic covariance matrix) than the estimated `SIGMA OF SINGLE DIFFERENCE OBSERVATION`. You may e.g. multiply the

formal rms values of the coordinates (see below) with the factor `SIGMA OF COORDINATE GROUP/SIGMA OF SINGLE DIFFERENCE OBSERVATION` to get more realistic rms values or you may derive an approximate value from the (unweighted) repeatability summary (block `COMPARISON OF STATION COORDINATES...`) or, as a third possibility, use the (weighted) rms values specified in block `MEAN VALUES OF GEOCENTRIC X,Y,Z - COORDINATES`.
In the case of free network solutions this coordinate group rms value may be too pessimistic, because the coordinate repeatabilities are computed without allowing for Helmert parameters. In that case you should use the second possibility to derive more realistic rms values for your coordinates.

`RESULTS OF COMBINED SOLUTION FOR STATION COORDINATES`
In this block a list of coordinate estimates is given in the following way (identical to GPSEST):

```
TOTAL NUMBER OF STATIONS:   103
NUM  STATION NAME    PARAMETER    A PRIORI VALUE       NEW VALUE   NEW-A PRIORI RMS ERROR
-----------------------------------------------------------------------------------------

413  QUIN 40433M004  X           -2517230.9430      -2517230.9590    -0.0160    0.0001
                     Y           -4198595.2572      -4198595.1676     0.0896    0.0001
                     Z            4076531.3765       4076531.2646    -0.1119    0.0001

                     HEIGHT          1105.8994          1105.7750    -0.1244    0.0001
                     LATITUDE     39 58 28.398327    39 58 28.396976  -0.0417    0.0001
                     LONGITUDE  -120 56 39.929157  -120 56 39.931674  -0.0596    0.0001
...  .............
```

The Table needs no further explanation.

In columns 100 to 132, in the lines corresponding to `HEIGHT`, `LATITUDE`, and `LONGITUDE` you find information concerning the error ellipsoids including some projections for each site.

```
RMS ERROR   3-D ELLIPSOID       2-D ELLIPSE
--------------------------------------------
0.0013      0.0013    0.3                           (a)
0.0002      0.0001   86.6       0.0001   86.7       (b)
0.0001      0.0002   -0.1       0.0002              (c)
 (1)         (2)      (3)        (4)      (5)
```

6 values (3 main axis parameters and 3 angles) characterize the three-dimensional ellipsoid and 3 values (2 main axis parameters and 1 angle) describe the two-dimensional error ellipse.

The lines are marked with (a), (b), and (c), the columns with (1) - (5). These marks are not included in the output file. The values (not extracted from the previous coordinate example) are defined as follows:

Column (1) : contains the rms errors in height (a), latitude (b), and longitude (c). These values are the intersections of the three-dimensional error ellipsoid with the coordinate axes of the local geodetic coordinate system.

Column (2) : contains the lengths in meters of the principal axes of the three-dimensional error ellipsoid.

Column (3) : contains in the first line (a) the zenith distance in degrees (°) of the longest axis (2a), in the second line (b) the azimuth, counted positive in the direction to east, of the axis (2b) and in the third line (c) the elevation angle of the same axis (2b).

Column (4) : contains the lengths in meters of the principal axes of the two-dimensional error ellipse in the horizontal plane.

Column (5) : contains the azimuth in degrees (°) of the principal axis (4b), counted positive in the direction to east, in the horizontal plane.

This information is useful to produce plots of error ellipses as shown in Figure 18.6. The shape of the error ellipses is typical for ambiguity-resolved regional solutions. Regional solutions without ambiguity resolution usually show larger rms values in the east-west direction by a factor of about 2.

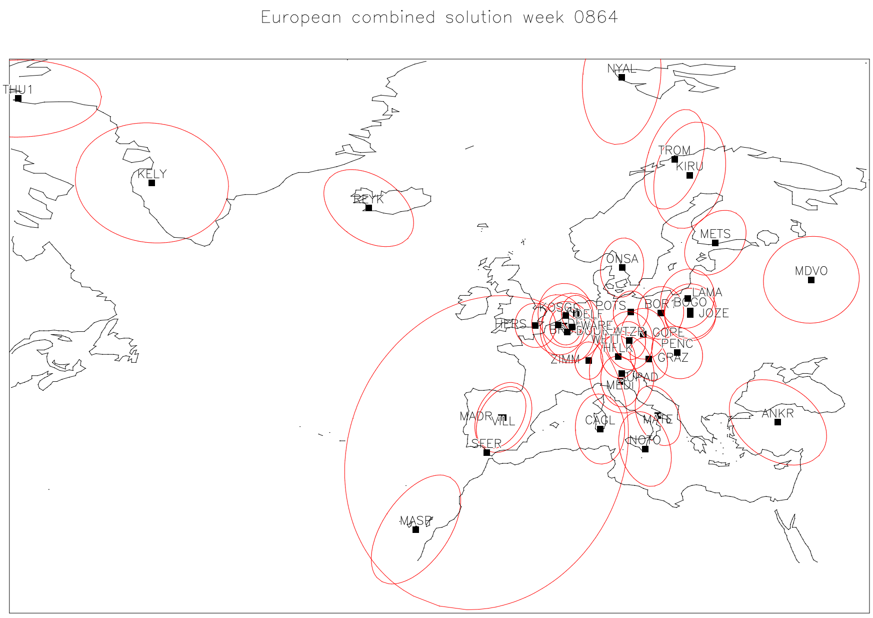


Figure 18.6: Error Ellipses using the Values of the GPSEST or ADDNEQ Program Output. The big Error Ellipse for the Site SFER is a Consequence of the small Number of Observations.

`Results for other parameter types`

The results for the other parameter types (e.g. troposphere parameters, orbits, etc.) are not explained in more detail, here. The information given in the program output should be sufficient.

`MEAN VALUES OF GEOCENTRIC X,Y,Z - COORDINATES`

In this block we summarize the coordinate estimates in the geocentric coordinate system together with the associated rms information. The formal rms of the previous block is repeated (`RMS1`). The formal rms error of the station position `RMS1-XYZ` $= \sqrt{\mathtt{RMS1}_X^2 + \mathtt{RMS1}_Y^2 + \mathtt{RMS1}_Z^2}$ is given in the last column. The second rms value `RMS2` is derived from weighted coordinate repeatabilities, similar to the computation of the group rms in the output block `SHORT SOLUTION STATISTICS`. In this case we consider one coordinate component as a group.

The `EPOCH` of the coordinates listed here and also in the resulting coordinate file (see Section 23.8.1) is usually the middle of the observation time span. Only in one case we refer the resulting coordinates to a different epoch: If you specify a special coordinate fixing file (see Section 18.7.4) without specifying a special velocity fixing file to propagate the coordinates to the middle observation epoch, we retain the epoch of the special coordinate fixing file as reference epoch for the resulting coordinates.

```
-------------------------------------------------------------------------------------------
MEAN VALUES OF GEOCENTRIC X,Y,Z - COORDINATES
RMS1: FORMAL ACCURACY OF EACH COORDINATE COMPONENT FROM COMBINED SOLUTION
RMS2: RMS OF WEIGHTED AVERAGE OF EACH COORDINATE COMPONENT
-------------------------------------------------------------------------------------------
EPOCH: 1996-07-24 11:59:45
VELOCITY MODEL INTRODUCED TO INDIVIDUAL SOLUTIONS:  ZERO VELOCITY FIELD


NUM  STATION      #FIL FLG     X (M)   RMS1  RMS2        Y (M)     RMS1  RMS2  ...RMS1-XYZ
-------------------------------------------------------------------------------------------

186  CAGL 12725M003  2 M 4893378.9431 0.0008 0.0015   772649.6258 0.0002 0.0004 ... 0.0011
155  MATE 12734M008B 3 M 4641949.7189 0.0007 0.0016  1393045.2699 0.0002 0.0023 ... 0.0010
122  MASP 31303M002  2 M 5439192.2475 0.0011 0.0031 -1522055.6314 0.0003 0.0038 ... 0.0013
... ..... .........    .  .
```

`LIST OF RMS VALUES`

Each NEQ system (corresponding to one NEQ file) is resolved individually applying the new specified input options. The list of rms values is useful to have a good overview about the impact of each contributing solution.

```
FILE  FILE NAME                           RMS     #OBS #PAR(ORIG.) #PAR (SOLVED)  MJD
-------------------------------------------------------------------------------------


  1   M:[COMB_GLO.OUT]EUR08577.NEQ     0.0038   838976     10226       114    50246.49983
  2   M:[COMB_GLO.OUT]EUR08587.NEQ     0.0024   870151      9936       114    50253.49983
  3   M:[COMB_GLO.OUT]EUR08597.NEQ     0.0035   899299      9733       111    50260.49983
  4   M:[COMB_GLO.OUT]EUR08607.NEQ     0.0036   884395      9959       114    50267.49983
  5   M:[COMB_GLO.OUT]EUR08617.NEQ     0.0036   768605     12046       114    50274.49983
  6   M:[COMB_GLO.OUT]EUR08627.NEQ     0.0034   838939     11874       114    50281.49983
  7   M:[COMB_GLO.OUT]EUR08637.NEQ     0.0036   833443     10256       114    50288.49983
  8   M:[COMB_GLO.OUT]EUR08647.NEQ     0.0037   915804     12007       114    50295.49983
-------------------------------------------------------------------------------------
                                       0.0037  6849612     85251       123    50256.99983
```

`FINAL LIST OF COVFACTOR VALUES`

This list shows the rescaling values. Values $\neq 1.0$ are possible by specifying a `.WGT` file (see Section 23.8.13).

`COMPARISON OF STATION COORDINATES...`

In this block we compare the results of the *site coordinates* for each individual solution with the combined solution. The *coordinate repeatabilities*, in units of mm, are given in the local north-east-up coordinate system. The intention is to detect outliers (see also block `OUTLIER DETECTION`). In the column `RMS` we compute an unweighted rms for the estimation of a single coordinate component (for more information see the explanations concerning block `SHORT SOLUTION STATISTICS`).

Specifying a `.PLT` file (see Section 23.8.10) saves the same information into a file in a different format, better suited as input to conventional plot tools.

We mentioned already in Section 18.3.4 that in the case of free network solutions the residuals are obtained after a Helmert transformation. In the case of long time series including velocity estimation you may wish to remove the effect caused by the estimated velocity. Edit the I-file, search for the string `REPET.PLT` and insert there a "2" instead of "0". The default plot option is "0" to retain the linear development of the coordinate components in the residuals.

```
-------------------------------------------------------------------------------------
COMPARISON OF STATION COORDINATES WITH RESPECT TO THE COMBINED SOLUTION IN MM
- UNWEIGHTED RMS OF INDIVIDUAL COORDINATE RESIDUALS
-------------------------------------------------------------------------------------

TOTAL NUMBER OF STATIONS:    41
-------------------------------------------------------------------------------------

NUM  STATION         #FIL C   RMS     1     2     3     4     5     6     7     8
-------------------------------------------------------------------------------------

186  CAGL 12725M003    8  N    1.2   -1.0  -0.7   0.7   0.1   0.3   2.4  -0.5   1.6
                          E    1.6   -0.7   1.2  -2.7  -0.3  -0.7   0.9   1.7   2.2
                          U    4.6   -1.7  -4.8   0.3  -5.8  -4.4   6.7   4.2   2.4

158  ZIMM 14001M004    8  N    1.7    3.3   0.0  -0.3  -1.6  -0.6   1.7   0.8  -1.7
                          E    1.7   -2.7  -0.9  -0.6   2.5   1.3  -1.6   0.0  -0.5
                          U    3.9    5.6   6.2   1.5   0.3  -3.9  -3.7   0.0  -1.6
...  ..............
```

`UNWEIGHTED RMS VALUES WITH RESPECT TO THE COMBINED SOLUTION`

This block contains a summary of the *repeatabilities for each file*. This information is well-suited to find out whether particular normal equations contain problems. Because unweighted rms values are computed it may occur, that outliers for a single site for a particular solution (e.g. due to a small number of observations), degrade the summary values for that specific solution.

`FAC` is the ratio of the repeatability and the formal rms value. The information contained in this line may also be saved to a `.WGT` file using Panel 4.8.1–0, option `COVARIANCE COMPON`. Furthermore, we use these values for the outlier detection described below.

```
TOTAL NUMBER OF STATIONS:    41
----------------------------------------------------------------------------------

                    #FIL C   RMS     1     2     3     4     5     6     7     8
----------------------------------------------------------------------------------

     COMBINATION       8  N    1.7    1.7   2.0   1.5   1.7   2.0   1.3   1.4   2.2
                          E    1.7    2.2   1.8   1.5   1.6   1.7   1.1   1.7   2.0
                          U    4.7    5.3   5.2   3.1   5.3   4.2   4.8   5.6   4.4
                       #STA     41     38    38    37    38    38    38    38    38
                        FAC          4.54  7.67  5.48  5.00  4.71  5.23  4.82  4.60
```

`WEIGHTED RMS VALUES WITH RESPECT TO THE COMBINED SOLUTION`

This block contains the same information as in the previous block, but now the rms values are computed using the formal rms values for each coordinate component estimation. Outliers due to a small number of observations do not play a major role, here. This information is available for free network solutions, only.

`OUTLIER DETECTION`

The residuals in the block `COMPARISON OF STATION COORDINATES...` are analysed for outliers using the information about the individual formal rms values. The "detection level" for the components `NORTH`, `EAST`, and `UP` is 3 times of the mean formal rms of all contributing files (the column `RMS` in block `UNWEIGHTED RMS VALUES WITH RESPECT...`). Outliers of several meters may disturb outlier detection. You may pre-eliminate sites of particular solutions using a station problem file (see Section 23.4.12).

```
---------------------------------------------------------------------------
OUTLIER DETECTION USING THE MEAN REPEATABILITY RMS OF EACH COMPONENT
DETECTION LEVEL (RESIDUUM/RMS): 3.00
NORTH:  0.006, EAST:  0.006, UP:  0.018 (M)
---------------------------------------------------------------------------
FILE  STATION           COMPONENT    RESIDUUM(M)     RMS(M) RMS*FAC(M)   GRP

   1  MATE 12734M008        U            -0.0205     0.0011     0.0062
   4  SFER 13402M004        N            -0.0056     0.0002     0.0013
   5  MATE 12734M008B       N            -0.0091     0.0002     0.0011
   5  MATE 12734M008B       E            -0.0068     0.0001     0.0008
   8  ANKR 20805M002        N            -0.0074     0.0002     0.0012
```

`UNWEIGHTED RMS VALUES OF THE COMPARISON BETWEEN THE SOLUTIONS`

Similar comparisons as those in block `UNWEIGHTED RMS VALUES WITH RESPECT...` are also performed between individual solutions (instead of comparing each solution with the combination). To reduce the amount of output we give this information only, if fewer than 16 normal equation files are processed.

```
TOTAL NUMBER OF STATIONS:    41
-----------------------------------------------------------------------------

                     FIL C          1     2     3     4     5     6     7
-----------------------------------------------------------------------------

                      2  N          1.9
                         E          1.7
                         U          6.1
                      #STA           38

                      3  N          2.0   2.1
                         E          2.4   2.0
                         U          6.8   6.1
                      #STA           37    37
                      4  N          2.8   2.5   2.0
                         E          3.2   2.5   2.3
                         U          8.2   6.1   6.1
                      #STA           37    37    37
                      ....         ....  ....  ....  ....
```

COMPARISON OF BASELINE LENGTHS

If you defined baselines using a (.BSL) file (see Section 23.8.25) in ADDNEQ you also get baseline statistics (residuals in latitude, longitude, height, and length) for those baselines. The relative error (expressed in PPM = parts per million) is computed as the ratio of the rms of the quantity considered (in column D(LGT)) and the baseline length. The RMS values are the formal rms values derived from the individual solutions. An example for the development of the baseline length and the associated formal rms errors (column (RMS)) is given in Figure 18.7. The velocity value given in the first line refers to the baseline length.

```
BASELINE : WETT 14201M009    TO  MATE 12734M008   VELOCITIES:-0.0035 +- 0.00003
-------------------------------------------------------------------------------
     FILE   BASE.LENGTH    D(LAT)    D(LON)    D(HGT)    D(LGT)    (PPM)  (RMS)
       1    989988.2166   -0.0051   -0.0167    0.0114    0.0091   0.0092 0.0005
       2    989988.2171   -0.0106   -0.0055    0.0031    0.0095   0.0096 0.0007
       3    989988.2235   -0.0154   -0.0026   -0.0283    0.0159   0.0161 0.0011
       4    989988.2176   -0.0076   -0.0062    0.0184    0.0101   0.0102 0.0007
    ...     ...........   .......   .......   .......   .......  ....... ......
     164    989988.2039    0.0021    0.0045   -0.0533   -0.0036  -0.0037 0.0003
     165    989988.2073   -0.0012    0.0035   -0.0410   -0.0003  -0.0003 0.0003
     166    989988.2075    0.0002    0.0025   -0.0239    0.0000   0.0000 0.0003
     167    989988.2120   -0.0060    0.0026   -0.0228    0.0045   0.0045 0.0003
-------------------------------------------------------------------------------
     164    989988.2076    0.0048    0.0054    0.0163    0.0051   0.0052 0.0005
```

18.8 Handling Parameters and NEQ Files in Programs GPSEST and ADDNEQ

Let us address in this section a few important aspects concerning the parameters and the NEQ files as they occur in programs GPSEST and ADDNEQ.

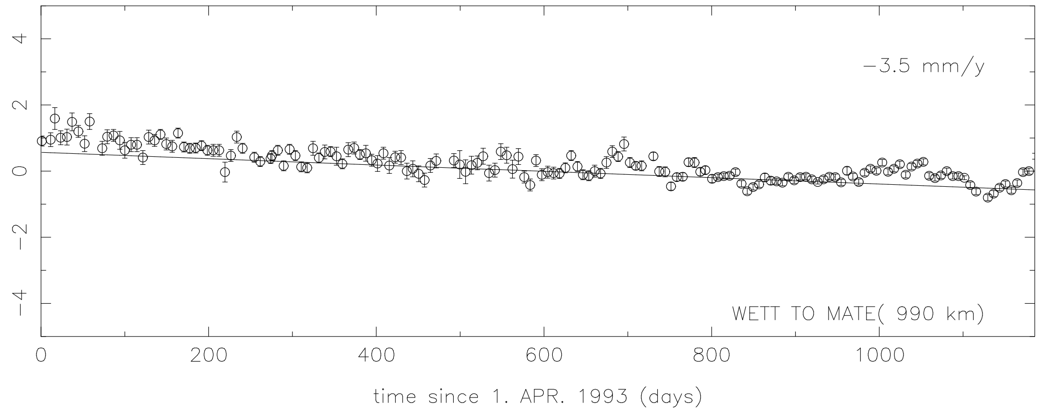


Figure 18.7: Plot of the Baseline Lengths Residuals and the Associated RMS Errors (in cm) Derived from the ADDNEQ (or COMPAR) Output.

GPSEST Do **not** use fix sites when saving normal equations, because GPSEST does not set up the coordinate parameters for fixed sites. Constrain the stations (using e.g. an a priori sigma of 0.0001 m) instead.

GPSEST Do not resolve ambiguities and create the NEQ file in the same program run. Solve ambiguities first, then introduce the resolved ambiguities in the second run and generate the NEQ file.

GPSEST Select pre-elimination of type `AI` for ambiguity parameters when saving the NEQs.

GPSEST Select pre-elimination of types `BI` or `AI` for troposphere, if you do not want to change the troposphere parameter handling later in ADDNEQ, select `NO` if you want to include them in the NEQs.

ADDNEQ In Panel 4.8.1–2, option `SITE-SPECIFIC TROPOSPHERE`, say `YES` to allow the specification of a priori sigmas (if you are not sure whether there are troposphere parameters in the NEQs).

ADDNEQ Fixing sites is allowed (as opposed to GPSEST), because fixing is equivalent to constrain coordinates with an a priori sigma of 0.001 mm in ADDNEQ.

ADDNEQ The generation of a SINEX file is not allowed (NEQ file is allowed), if free network solutions are generated because of a singular a priori covariance matrix. The use of the inverted a priori covariance matrix would solve the problem, but this information is not supported by most of the SINEX reading routines.

ADDNEQ Saving a NEQ file is not possible (SINEX file is possible), if you specify a special coordinate file for fixed sites in Panel 4.8.1. This limitation is not important, because you may create normal equations containing all information without this special file.

ADDNEQ Saving a NEQ file is not possible (SINEX file is possible) if velocities are solved for. ADDNEQ supports the estimation of velocities, but not the handling and combination of estimated coordinates **and** velocities.