Space Geodesy

Definitions and Principles

Geodesy is the science studying the size and the figure of the Earth including the determination of the Earth's gravity field. *Geodetic astronomy* is that part of astronomy dealing with the definition and realization of a terrestrial and a celestial reference frame (cf. TERRESTRIAL COORDINATE SYSTEMS & FRAMES). By *Space Geodesy* we mean, then, those aspects of geodesy and geodetic astronomy studied by using natural or artificial celestial bodies as observed objects or as observing platforms. In the older literature the term *Cosmic Geodesy* is sometimes used as a synonym. Space geodesy is thus defined through the observation techniques, below referred to as space geodetic techniques, or methods.

Space geodesy evolved rapidly in the second half of the twentieth century. The *space age* was initiated by the launch of the first artificial satellite, Sputnik I, on October 4 of the International Geophysical Year 1957. In the space age it became possible to deploy and use artificial satellites *either* to study size and figure of the Earth from space *or* to observe them as targets from the surface of the Earth. The use of artificial Earth satellites for geodetic purposes is also referred to as *satellite geodesy.* The second essential development consists of the *Very Long Baseline Interferometry* (VLBI) technique as a new tool to realize an extraordinarily accurate and stable inertial reference system and to monitor Earth rotation using quasars (cf. EXTRAGALACTIC REFERENCE FRAMES).

Today, space geodetic techniques are the primary tools to study size, figure and deformation of the Earth, and its motion as a finite body in the inertial reference system (cf. SPACE&TIME REFERENCES: CONCEPTS). Space geodetic techniques thus are the fundamental tools for geodesy, geodetic astronomy, and geodynamics.

Space geodetic observations contain information about the position (and motion) of the observed object and the observer. Therefore, space geodetic observations also contain information concerning the transformation between the terrestrial and the inertial systems. The Earth orientation parameters, *i.e.*, polar motion, UT1, precession and nutation (cf. EARTH ROTATION: THEORY, POLAR MOTION AND LENGTH OF DAY) define this transformation.

The role of the Earth's atmosphere

In space geodesy the signals of the observed or observing celestial bodies have to cross the Earth's atmosphere. This changes the path and the travel times of the signals. These are referred to as *refraction effects*. Refraction is usually considered a nuisance in astronomy, geodesy and geodynamics — as a matter of fact it is the motivation for many spaceborne experiments related to this field of science. In recent years refraction effects are more and more considered and understood as a primary source of information for atmosphere science and are monitored through space geodetic methods. Let us point out that the same signals and space geodetic analysis methods are used to study the Earth's atmosphere as for geodetic and geodynamics purposes. Interdisciplinary studies and projects have become important aspects in modern space geodesy.

Whether the atmosphere related signal is useful depends on the wavelengths of the analyzed signals. If we measure, *e.g.*, distances or distance differences to satellites using optical signals, refraction effects may be computed with sub-centimeter accuracy using pressure, temperature and humidity registrations at the observing sites. We may therefore conclude, *e.g.*, that laser ranging is not capable of contributing to atmosphere monitoring. This fact may also be formulated in the positive way: Laser observations are well suited for calibrating other techniques, which are more prone to atmospheric effects.

For microwave techniques (Doppler, GPS, VLBI) we have to distinguish between ionospheric refraction stemming from the ionized upper part of the atmosphere (extending up to about 1500km) and tropospheric refraction, stemming from the lower, neutral layers of the atmosphere. Ionospheric refraction is wavelength-dependent and may be (almost completely) eliminated if coherent signals are sent through the atmosphere on different carrier wavelengths. In the VLBI technique this is achieved by observing the quasars in different wavelengths, in the Doppler or GPS technique the same is achieved by using two different wavelengths for signal transmission.

For microwave techniques tropospheric refraction is the ultimate accuracy-limiting component in the error budget. As opposed to range observations in the optical band, we have to take into account the so-called "wet component" of tropospheric refraction, which is highly variable in time and space. This fact forces analysts using microwave observations to introduce station and time specific parameters (or to model the effect as a random process). It allows, on the other hand, analysts to determine the water vapor content above an observatory with high accuracy and high temporal resolution (Bevis et al., 1992).

Optical period

For centuries optical (astrometric) observations were the only observation type available in astronomy. In the pre-space era a series of astrometric instruments was used for the purpose of defining a terrestrial reference frame and for monitoring Earth rotation. The photographic zenith tube and the *Danjon astrolabe* were probably the most advanced of these instruments. They were widely used by observatories contributing to the International Polar Motion Service (IPMS) and the Bureau International de l'Heure (BIH) to determine the geographic latitude of a station with a precision of about 10-40mas (milliarcseconds) in one night. We refer to (Moritz and Mueller, 1988) for more information.

Optical observations where already made of the first generation of artificial Earth satellites, like Sputnik 2 and Explorer 1. The balloon satellites Echo 1 and 2 and PAGEOS (passive geodetic satellite), which could even be seen "by naked eye", were observed by a worldwide optical tracking network. These satellites were (supposedly) spherical, consisted of layers of aluminized mylar foil, and, thanks to their brightness, their tracks could easily be photographed against the star background. It was not trivial to assign time-tags to specific points of the track. Much better suited from this point of view, although more difficult to track, were smaller satellites like Geos 1 (Explorer 29) and Geos 2 (Explorer 36) equipped with flash lamps allowing for tens of thousands of high-precision optical observations. Obviously, the quasi-simultaneity of observations from different sites was possible.

Fascinating results came out of this first phase of satellite geodesy. The geodetic datums on different continents could be related to the geocenter and thus to each other with an accuracy of about 5m. First reliable coefficients of the gravity field (spherical expansion up to degree and order 12-15) could be also derived.

The astrometric technique, when applied to artificial satellites in the 1960s and 1970s, had serious disadvantages. The star catalogues were not of sufficiently good quality and the processing time (time between observation and availability of results) was of the order of a few weeks in the best case. This, and the advent of new observation techniques promising higher accuracy, actually ruled out astrometric techniques for a number of important applications. The optical technique no longer played a significant role in space geodesy after about 1975.

In view of newly developed observation techniques (CCD, Charge Coupled Device techniques (cf. OBSERVATION TECHNIQUES)) and much better star catalogues based on astrometry missions (*e.g.*, HIPPARCOS mission, (cf. HIPPARCOS)) it may well be that optical observations will again play a role in space geodesy in the future.

Doppler period

The U.S. Navy Navigation Satellite System (NNSS), also called TRANSIT system after the survey transit instrument, had a significant impact on the development of space geodesy. It proved that a system based on the measurement of the Doppler shift of a signal generated by a stable oscillator on board a satellite could be used for relative positioning with remarkably high accuracies (0.1-0.5m relative, about 1m geocentric). The satellites sent information on two carrier frequencies (400Mhz and 150MHz) near the microwave band.

The two frequencies allowed for a compensation of ionospheric refraction. Rather small receivers connected to omni-directional antennas made the technique well suited to establish regional or even global geodetic networks. Observation periods of a few days were required to obtain the above mentioned accuracy.

The NNSS satellites were in polar, almost circular, orbits about 1100km above the Earth's surface. Only one satellite at a time could be observed by one receiver. As opposed to astrometry the Doppler technique was weather-independent. Until a significant part of the Global Positioning System (GPS) was deployed (around 1990) the NNSS played a significant role in space geodesy. Many Doppler campaigns were organized to establish local, regional or global networks. With the full deployment of the GPS in the 1990s the geodetic community eventually lost interest in the Doppler system. The Transit system was shut down as as a positioning system in December 1996 but continued operating as an ionospheric monitoring tool. For more information concerning the Doppler system we refer to (Kouba, 1983).

Satellite and Lunar Laser Ranging (SLR and LLR)

Laser stands for *Light Amplification through Stimulated Emission of Radiation*. The laser technique, developed in the 1950s, is able to generate high energetic short light pulses (of a few tens of picoseconds (ps) (1 ps=10^{-12}s)). These pulses are sent out by a conventional astronomical telescope, travel to the satellite (or the Moon), are reflected by special corner cubes (comparable to the rear reflectors of bicycles) on the satellite (Moon) back to the telescope, where they are detected. The measurement is the travel time Δt of the laser pulse from the telescope to the satellite and back to the telescope. Apart from refraction this light travel time, after multiplication with the speed of light c in vacuum, equals twice the distance ρ_r^s between satellite and telescope at the time the light pulse is reflected from the satellite $\rho_r^s \approx \Delta t \cdot c/2$. Today's Satellite Laser Ranging (SLR) technique is used to determine the "true" distances between observatories and satellites with an accuracy of a few millimeters and, if required, with a high repetition rate (several times per second).

SLR techniques may be used for every satellite equipped with corner cubes. Fig. 1 shows Lageos II, a typical SLR-dedicated satellite which was launched in 1992. Lageos II is a spherical satellite with a diameter of 0.6m, a weight of 405kg. 426 corner cubes are inlaid in its surface. Lageos II is a close relative of Lageos I, which was launched in 1976. The two Lageos satellites are in stable, almost circular orbits about 6000km above the surface of the Earth.

The two Lageos satellites are primary scientific tracking targets for the *International Laser Ranging Service (ILRS)*. The two satellites have contributed in a significant way to the determination of the Earth's gravity field. Many more targets are regularly observed by the ILRS. Some, like the French low orbiting satellite *Starlette*, with a diameter of 24cm, are similar in design to the Lageos satellites and serve a similar purpose. For others SLR is just the primary or backup technique for precise orbit determination.

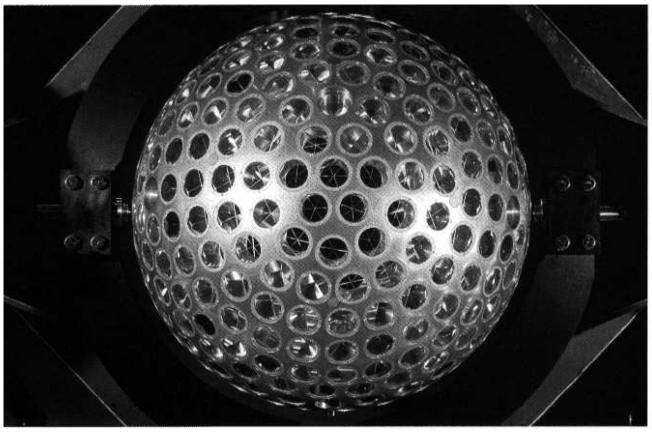

Figure 1: The Lageos II Spacecraft

With the exception of UT1, the SLR technique is able of determining all parameters of geodetic interest (station coordinates and motion, Earth rotation parameters, gravity field). The unique and most valuable contributions lie in the determination of the Earth's (variable) gravity field, in the determination of the geocenter (*i.e.*, the location of the polyhedron formed by connecting the SLR stations with respect to the geocenter), and in calibrating geodetic microwave techniques.

From the technique point of view there is no principal difference between SLR and LLR (Lunar Laser Ranging): Light travel times are measured from the observatory to one of the laser reflectors deployed on the Moon by the Apollo space missions or the Russian unmanned Lunokhod missions. The scientific impact of LLR is significant. LLR was, *e.g.*, capable of measuring directly the secular increase of the Earth-Moon distance (3.8cm per year), an effect which is in turn coupled with the deceleration of the angular velocity of Earth rotation. Also, LLR is well suited to evaluate gravitational theories (cf. REFERENCE FRAMES & TIMESCALES IN GENERAL RELATIVITY).

Very Long Baseline Interferometry

Very Long Baseline Interferometry (VLBI) is the only non-satellite geodetic technique contributing data to the International Earth Rotation service (IERS). Its main features are discussed in (cf. EXTRAGALACTIC REFERENCE FRAMES).

Its unique and fundamental contribution to geodesy and astronomy consists of the realization of the inertial reference system and in the maintenance of the long-term stability of the transformation between the celestial and terrestrial reference frame.

The ICRS, International Celestial Reference System, was defined by the International Earth Rotation Service (Arias et. al., 1995). It was adopted by the International Astronomical Union (IAU) as the primary celestial reference system replacing the optical predecessors.

An accurate and stable celestial reference frame is a prerequisite for a terrestrial reference system. In this sense VLBI plays a decisive role in the definition of the terrestrial reference system, and in establishing the transformation between the two systems. In particular, VLBI is the only technique providing the difference UT1-UTC, *i.e.*, the difference between Earth rotation time and atomic time with state-of-the-art accuracy and excellent long-term stability. Also, VLBI is the only technique capable of determining precession and nutation with an angular resolution below the milliarcsecond level.

The observation and analysis aspects are today coordinated by the IVS, the International VLBI Service (Table 1).

The Global Positioning System (GPS)

GPS is probably the best known space geodetic technique, today. The system has an impact on science and society reaching far beyond space geodesy. GPS revolutionized surveying, timing, car and aircraft navigation. Virtually millions of hand-held receivers are in use today. Spaceborne applications of the GPS have a deep impact on geodesy and atmosphere sciences.

GPS is a navigation system allowing for instantaneous, real-time, "absolute" positioning on or near the surface of the Earth with an accuracy of a few meters. An unlimited number of users may use the system simultaneously. "Absolute" means that the estimated coordinates may be established using only one receiver and that they refer to a geocentric Earth-fixed coordinate system. This coordinate system, the WGS-84 (World Geodetic System), is today aligned with sub-meter accuracy to the ITRF, the International Terrestrial Reference Frame maintained by the IERS (cf. TERRESTRIAL COORDINATE SYSTEMS & FRAMES)).

The space segment of GPS nominally consists of 24 satellites (21 operational satellites plus 3 active spares). The satellites are in almost circular orbits distributed in six planes approximately 20,000km above the Earth's surface. These planes are separated by 60 deg on the equator and inclined by 55 deg with respect to the equator. The revolution period is half a sidereal day ($11^h 58^m$), which means that for a given location on the Earth's surface the satellite constellation above horizon repeats itself after one sidereal day (solar day minus four minutes). Fig. 2 shows the Block II satellite.

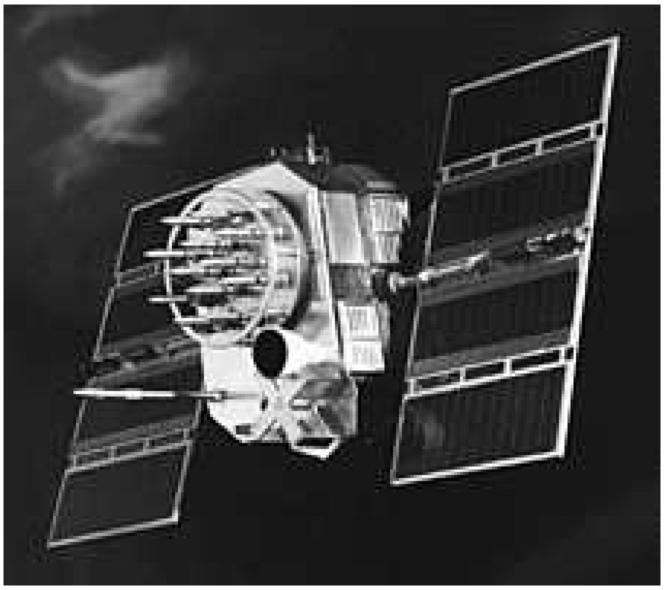

Figure 2: Block II Satellite

This is the satellite with which the first full

GPS generation was built. We distinguish the main body of the satellite with the antenna array pointing to the center of the Earth and the solar panels. The attitude is maintained by *momentum wheels*, which have to guarantee that the antenna array is always pointing to the center of the Earth and that the solar panel axes are perpendicular to the Sun-satellite direction. The satellite is then capable of rotating the solar panels into a position perpendicular to the same direction.

Each satellite broadcasts on two carrier frequencies L1 and L2 of 1.57542GHz and 1.22760GHz respectively, corresponding to wavelengths of $\lambda_1 \approx$ 19 cm and $\lambda_2 \approx 24$ cm. Two types of codes are sent out, allowing the users to reconstruct the so-called *pseudorange* p_r^s (definition in eqn. (1)).

The pseudorange p_r^s and the geometric distance ρ_r^s between the satellite at signal emission time and the receiver at signal reception time are related through

$$p_r^s = \rho_r^s + c \cdot (\Delta t_r - \Delta t^s) + \Delta\rho_{atm}, \qquad (1)$$

where Δt^s is the error of the satellite clock with respect to the "true" (or system) time, Δt_r is the receiver clock error. $\Delta\rho_{atm}$ is the correction of the light travel time due to the atmosphere (sum of ionospheric and tropospheric refraction).

In addition, and among other important information, *broadcast orbits* allowing computation of the satellite position at emission time and *satellite clock corrections* mitigating the satellite synchronization error Δt^s *w.r.t.* the "true" or system time are sent out continuously using the phase modulation technique on L1 and L2.

Two kinds of code have to be distinguished, the so-called C/A-code (coarse acquisition code) allowing an accuracy of about 3m, and the P-code (precision code) allowing an accuracy of about 0.3m. Modern digital receivers show even a much better performance. The P-code currently is only available to privileged (*i.e., U.S. Department of Defense authorized*) user. The C/A code is transmitted on L1, an encrypted version of the P-code on both carriers. The broadcast orbits usually have an accuracy of about 3m. The satellite clock information is made available to the non-authorized user only with moderate accuracy, which limits real-time "absolute" positioning accuracy to about 100m.

Let us briefly address the principles of *absolute GPS positioning and navigation*: A GPS receiver is simultaneously observing several satellites (ideally all that are above the horizon). Using broadcast orbits to compute the satellite positions in the Earth-fixed system, standard atmosphere models to account for refraction, and the broadcast clock information to adjust the satellite clocks to system time, we are left with only four unknowns in eqn. (1), namely the three coordinates of the receiver position and the receiver clock error Δt_r – provided we consider only simultaneous observations. It is thus necessary and sufficient that a receiver tracks simultaneously four satellites in order to compute an instantaneous position (and a receiver clock error) with an accuracy of a few meters or about 100m for users having no access to P-code, respectively. The GPS constellation was designed, as a matter of fact, to ensure that (to the extent possible) four or more satellites are available all the time at each location on the surface of the Earth.

For scientific purposes the *phase observation* plays a decisive role. It is in essence identical with the so-called accumulated Doppler observation of the NNSS and it is closely related to the GPS code observation, as well. From the mathematical point of view, there are "only" two essential differences between phase and code, namely the much higher measurement accuracy of phase (millimeters rather than meters), and an additional unknown, the initial phase ambiguity parameter N_r^s, *per satellite pass*. All GPS receivers used for high accuracy geodetic applications record the phase observations on both carriers L1 and L2, in addition to the code.

The phase observations yields local GPS networks with mm-accuracy, regional and global networks with about cm-accuracy. This is only possible, if precise satellite orbit and clock information, such as generated by the *International GPS Service (IGS)*, is available. Fig. 3 shows the IGS network as of October 1998.

Over 200 IGS sites, distributed all over the globe permanently observe all satellites in view, transmit their observations (at least) on a daily basis to IGS Data Centers.

The data are then analyzed by IGS Analysis Centers, which deliver *rapid* and *final* products. Rapid IGS products are available with a delay of about one day, final products with a delay of about eleven days. Daily products include satellite orbits with an accuracy of about 0.05-0.1m, satellite clocks with an accuracy of about 0.3ns (nanoseconds), daily values of polar motion components accurate to about 0.1 mas (milliarcseconds), corresponding to 3mm on the Earth's surface), and length of day (lod) estimates with an accuracy of about 30μs/day. These products are essential contributions to the monitoring of Earth rotation (cf POLAR MOTION AND LENGTH OF DAY).

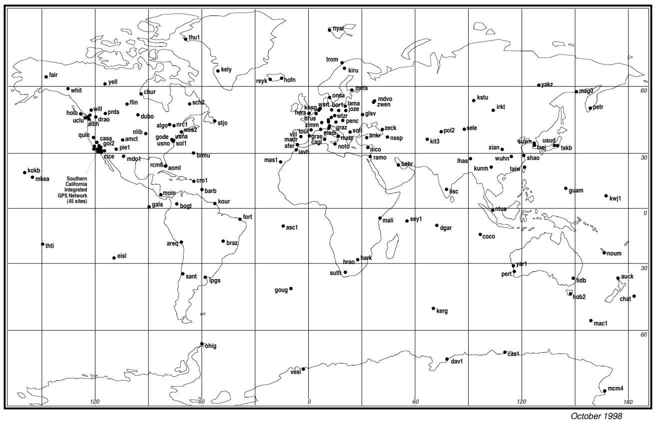

Figure 3: The International GPS Service Network

In addition the IGS Analysis centers perform weekly global coordinate solutions of their portion of the IGS network. These results are used, together with the results of the other space techniques, for the establishment of the International Terrestrial Reference Frame (cf. TERRESTRIAL COORDINATE SYSTEMS & FRAMES).

The IGS products (orbits, Earth rotation parameters, coordinates and velocities of IGS stations) are used as known *a priori* information to establish regional networks for crustal deformation studies (*e.g.*, the Californian SCIGN (http://www.scign.org), the Japanese 1000-receiver network for Earthquake monitoring) or for regional reference frame establishment and maintenance (*e.g.*, the EUREF network http://www. oma.be:/KSB-ORB/EUREF/ eurefhome.html, or the South American SIRGAS network http://www.dgfi.badw-muenchen.de/gps/sirgas.html).

More and more, the IGS network is used for purposes other than space geodesy. Let us mention that the IGS network has been enhanced to include time and frequency transfer and that it is able to monitor the ionosphere. For more information concerning the IGS and its interdisciplinary impact we refer to (Beutler et al., 1999).

From the point of view of space geodesy GPS is a "work horse" with important contributions to the establishment and maintenance of a dense terrestrial reference frame, and which provides Earth rotation parameters with a high time resolution. It should not be forgotten, that the GPS — like every satellite geodetic method — is not able to maintain a long-term stability of $UT1$ or of precession and nutation. Moreover, despite the fact that GPS is a satellite geodetic technique, it is not well suited to determine the Earth's gravity field or the motion of its station polyhedron *w.r.t.* the geocenter. The height of the GPS satellites is one of the limiting factors.

For more information concerning GPS as a tool for geodesy and geodynamics we refer to (Teunissen and Kleusberg, 1998).

Other Satellite microwave techniques

The Russian GLONASS (Global Navigation Satellite System) is so closely related to the GPS that there are a number of combined GPS and GLONASS receivers available. These receivers were used in the first global GLONASS tracking and analysis campaign, the *IGEX-98* (International GLONASS Experiment 1998). The experiment revealed that a combined analysis of GPS and GLONASS is very promising for science and navigation.

The French DORIS system (Doppler Orbitography by Radiopositioning Integrated on Satellite) proved to be a very powerful tool for orbit determination. It is, *e.g.*, one of the orbit determination system used in the TOPEX/Poseidon mission (see below). Also, DORIS possesses a very well designed ground tracking network. This is one reason why DORIS was accepted as an official space technique by the IERS (see Table 1).

The German PRARE (Precise Range and Range-rate Equipment) system may be viewed as the German counterpart of the DORIS system. It is used as an orbit determination tool, *e.g.*, on the European Space Agency's ERS-2 (Earth Remote Sensing) spacecraft.

Satellite Missions

There were many satellite missions in the past and there will be more in the future in which the satellite is used as an observing platform to study aspects of the Earth relevant to geodesy and geodynamics. Let us mention in particular that *altimetry missions* significantly improved our knowledge of the sea surface topography, ocean currents, tidal motions of the oceans, *etc.*

Fig. 4 shows the TOPEX/Poseidon spacecraft. The mission is a combined U.S. and French altimetry mission. It is actually the first mission which was specially designed to investigate ocean currents. One entire volume of the Journal of Geophysical Research was devoted to this mission (JGR, 1994).

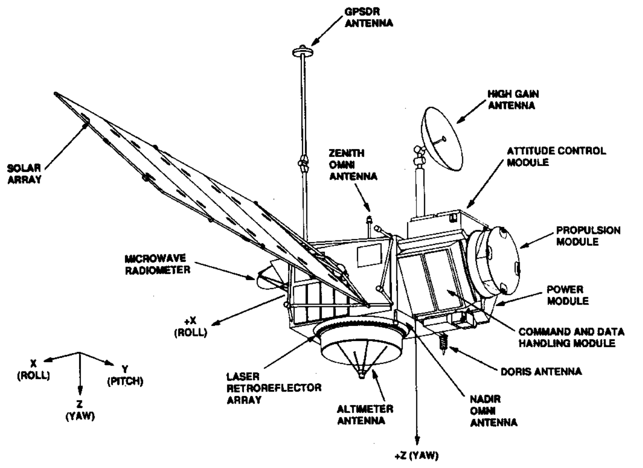

Figure 4: The TOPEX/Poseidon Spacecraft

For space geodesy the TOPEX/Poseidon mission was a kind of “rosetta stone mission” because its orbit was determined using three independent systems, the French DORIS system, SLR tracking, and the GPS. All three systems proved their capability. The radial component of the orbit (which is of crucial importance for altimetry missions) could be established with an accuracy of a few centimeters. Let us mention that TOPEX/Poseidon was neither the first, nor will it be the last altimetry mission.

For geodesy, geodynamics, and atmosphere physics the upcoming missions CHAMP (Challenging Mini-Satellite Payload for Geophysical Research and application, German mission), GRACE (Gravity Recovery and Climate Experiment, U.S./German mission), and GOCE (Gravity field and Ocean Current Explorer, ESA mission) are fascinating. It is expected that our knowledge of the gravity field (using spaceborne GPS receivers, accelerometers, or gradiometers) to measure the non-gravitational forces *resp.* gravity gradients will significantly increase through such missions.

Also, CHAMP, GRACE and GOCE are able to produce atmosphere profiles using the *occultation method*: the signal (phase and code) of a GPS satellite is monitored by a spaceborne GPS receiver on a low Earth orbiter (LEO) during the time period the line of sight LEO-GPS satellite scans through the Earth’s atmosphere. These developments support our initial statement that interdisciplinary aspects are becoming more and more important in Space Geodesy.

Organizations

Table 1 gives an overview of the institutions relevant for space geodesy.

They all are IAG (International Association of Geodesy) services. The IERS and the IVS are in addition IAU services (International Astronomical Union). The IERS and the IGS are members of FAGS (Federation of Astronomical and Geodesical Data Analysis Services).

IGS, ILRS and IVS are technique-specific services. The IERS is a multi-technique service. It was established in 1988 as successor of the International Polar Motion Service (IPMS) and the Earth rotation branch of the Bureau International de l’Heure (BIH). The IERS products are based on the products of the technique-specific services.

CSTG is a commission of IAG and a subcommission of COSPAR (Commission on Space Research). It has a coordinating function within space geodesy. In the time period 1995-1999 it was, *e.g.*, responsible for creating the ILRS and the IVS, and it organized the first global GLONASS experiment IGEX-98.

More information about these services may be found at the internet addresses in Table 1.

Acronym	Name, Mission, Internet
CSTG	Commission on International Coordination of Space Techniques. Coordination between space geodetic organizations, organize projects.
IERS	International Earth Rotation Service. Establish and maintain celestial and terrestrial reference frame, generate combined Earth orientation parameter series http://hpiers.obspm.fr
IGS	International GPS Service. Make available GPS data from its global network, producing and disseminating high accuracy GPS orbits, Earth rotation parameters, station coordinates, atmospheric information, etc. http://igscb.jpl.nasa.gov
ILRS	International Laser Ranging Service. Collects, archives, and distributes SLR and LLR datasets. Generates scientific and operational products http://ilrs.gsfc.nasa.gov
IVS	International VLBI Service for Geodesy and Astrometry. Operate or support VLBI programs. Organize geodetic, astrometric, geophysical research and operational activities. http://ivscc.gsfc.nasa.gov

Table 1: Space Geodetic Services

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