BASICS OF GPS DATA PROCESSING

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GPS/GLONASS SYSTEM DESCRIPTION

GPS/GLONASS SEGMENTS

NAVSTAR-GPS: **NAV**igation **S**ystem with **T**ime **A**nd **R**anging – **G**lobal **P**ositioning **S**ystem

GLONASS: **GLO**bal **NA**vigation **S**atellite **S**ystem (Russian pendant)

The GPS (and the GLONASS) consist of 3 main segments:

- **Space Segment**: the constellation of satellites
- **Control Segment**: operation and monitoring of the GPS/GLONASS System
- **User Segment**: all GPS/GLONASS receivers and processing software

We might add a 4th segment:

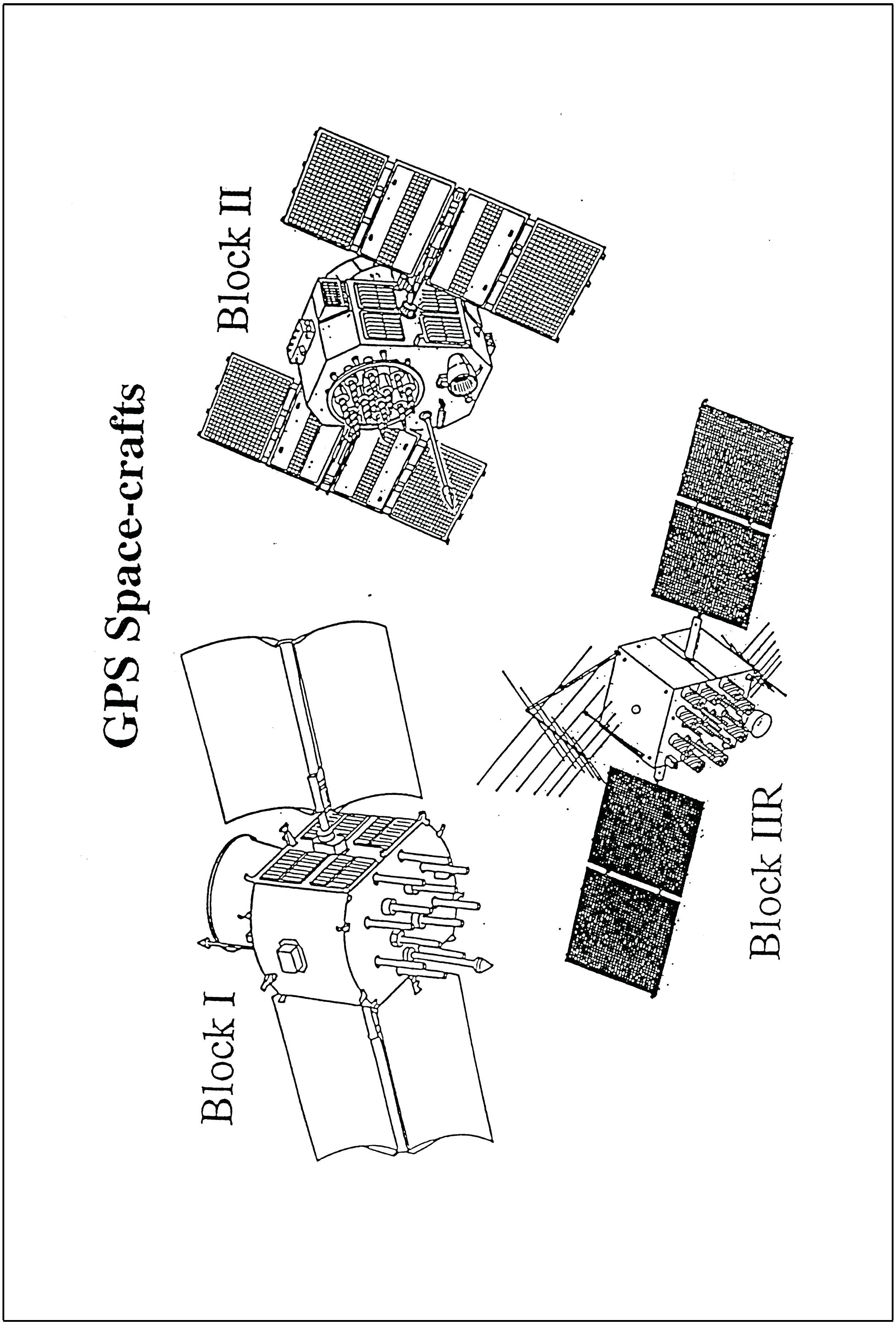
- **Ground Segment**: Civilian permanent networks of reference sites

SPACE SEGMENT

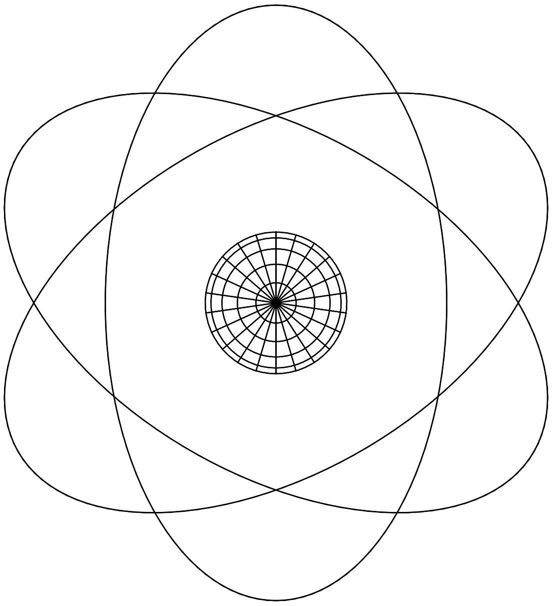
- The space segments nominally consist of **24 satellites**, presently:
  - **28** active GPS satellites
  - **11** active GLONASS satellites
- **Constellation design**: at least **4 satellites** in view from **any location** on the Earth at **any time**
- **Orbit characteristics**:

Characteristic	GPS	GLONASS
Semi-Major Axis (Radius)	26’600 km	25’500 km
Orbital Period	11 h 58 min	11 h 16 min
Orbit Inclination	55 deg	65 deg
Number of Planes	6 (60 deg spacing)	3 (120 deg spacing)
Number of Satellites	24 (3 spares)	24 (3 spares)
Approximate Mass	815 kg	700 kg
Data Rate	50 bit/sec	50 bit/sec
PRN Codes	Satellite-dependent	Satellite-indep.
Frequencies	Satellite-indep.	Satellite-dependent

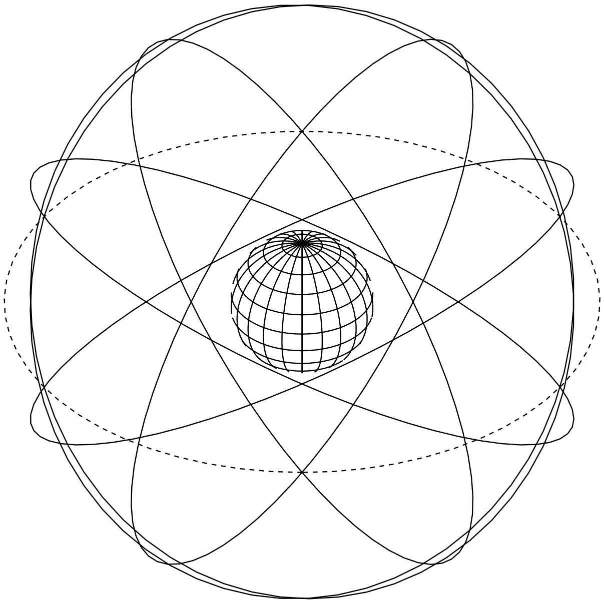
PRN Codes: Pseudo-Random Noise Codes.


GPS Space-crafts
Block I
Block II
Block IIR

GPS SATELLITE CONSTELLATION



GPS orbits viewed from latitude $\beta = 90^{\circ}$

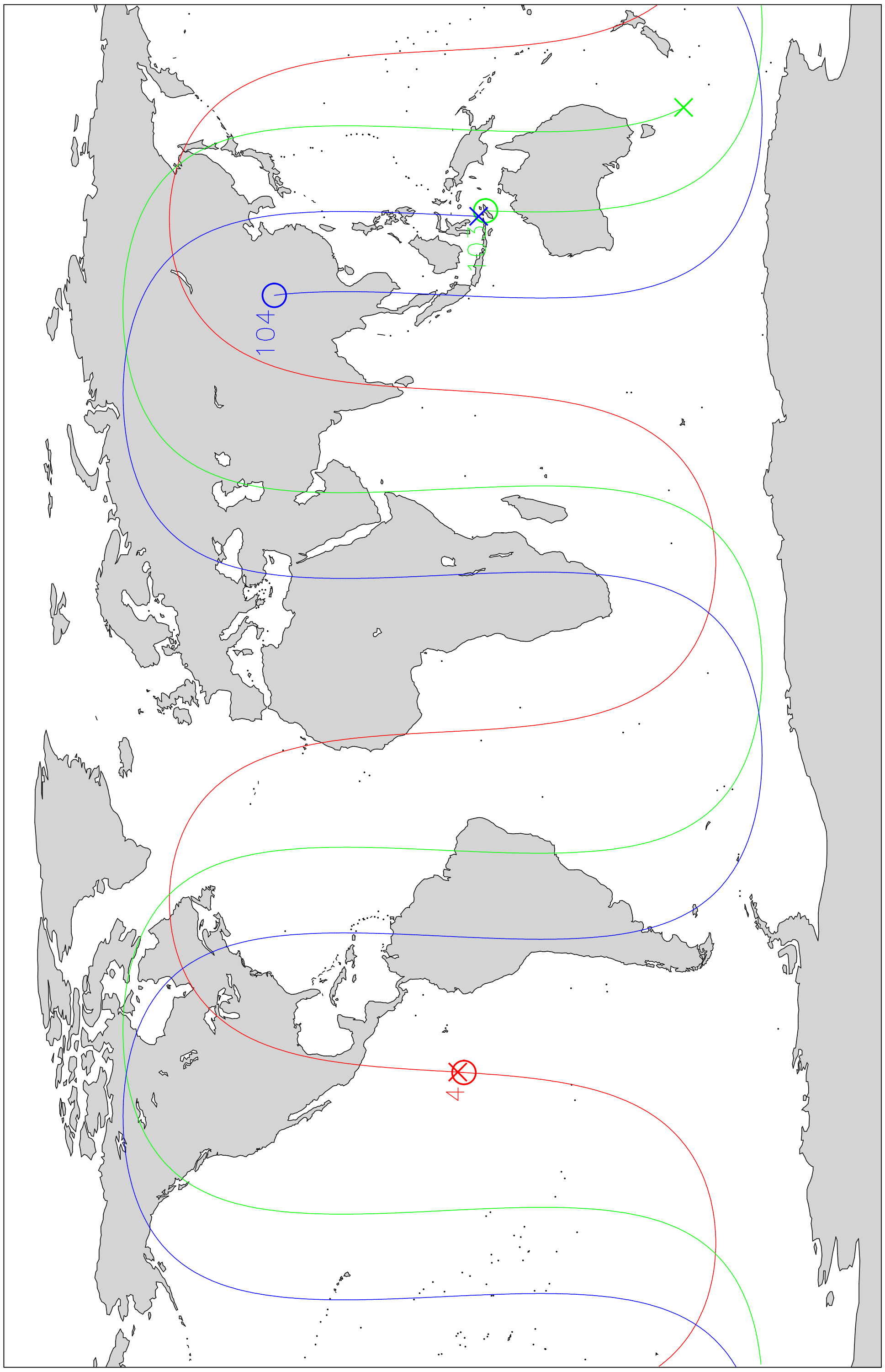


GPS orbits viewed from latitude $\beta = 35^{\circ}$

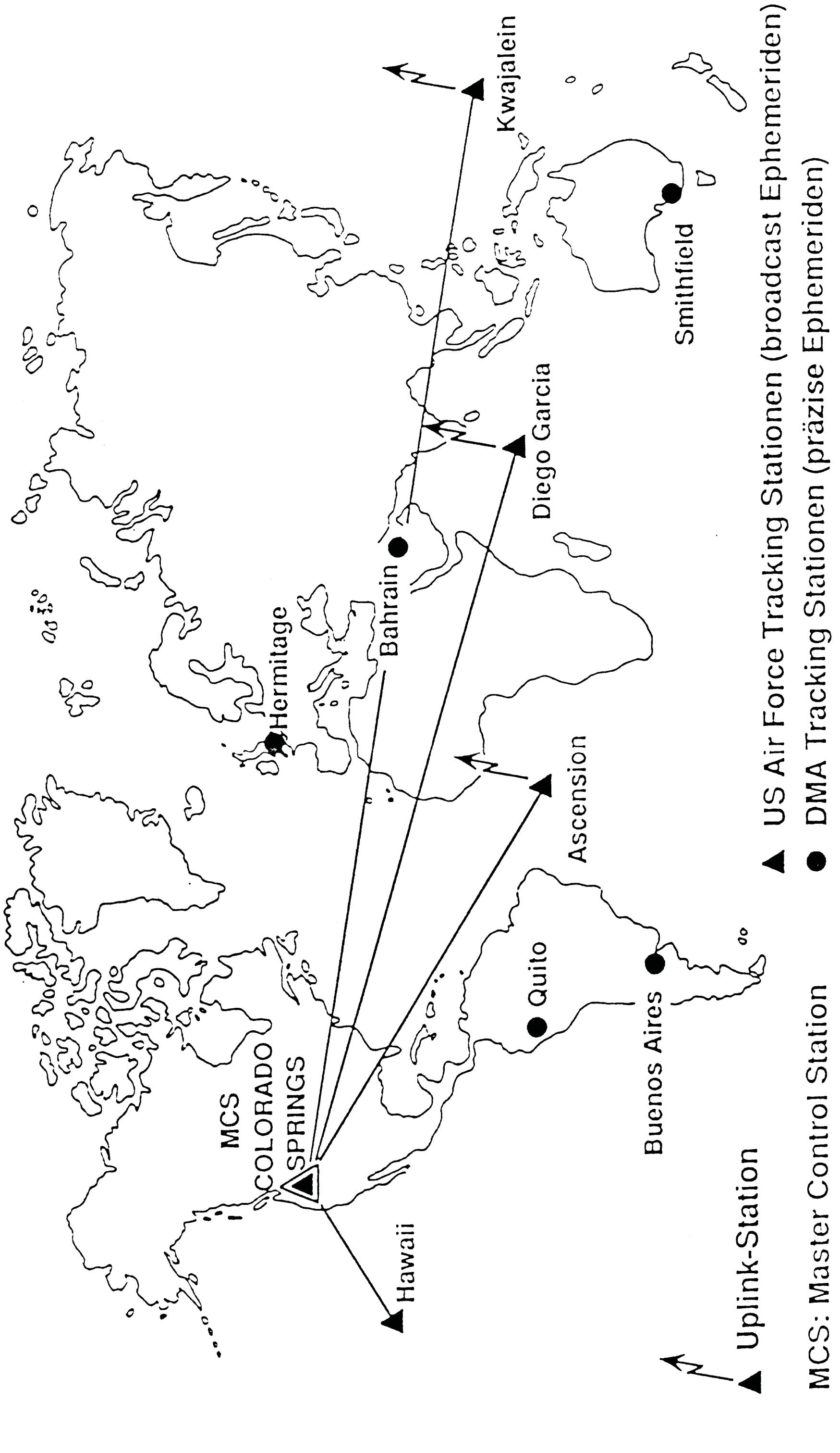
ORBIT CHARACTERISTICS (CONT.)

- **Speed** of satellites relative to the Earth center approximately **4 km/s**, relative to the user up to **2.8 km/s**.
- GPS satellites repeat their ground tracks after: **1 sidereal day** = 23 h 56 min = 2 orbital periods. In solar days (UT) the same geometry is reached 4 minutes earlier every day.
- GLONASS satellites repeat their ground tracks after: **8 sidereal days** = 17 orbital periods. Identical geometry (but realized with different satellites) after 1 sidereal day if 8 satellites equally distributed in each plane.
- **2:1 commensurability** of GPS revolution period with Earth rotation leads to **resonance effects** with gravity field (many manoeuvres necessary); not the case for GLONASS.
- Better satellite visibility of GLONASS satellites from **polar regions** (larger orbit inclination).

SATELLITE GROUND TRACKS



CONTROL SEGMENT



USER AND GROUND SEGMENT

User Segment:

- All GPS/GLONASS receivers on land, on sea, in the air and in space.
- Extremely broad user community with applications of the GPS/GLONASS for
  - Navigation
  - Surveying
  - Geodynamics and geophysics
  - Remote sensing (troposphere and ionosphere)
  - Time and frequency transfer
  - ...

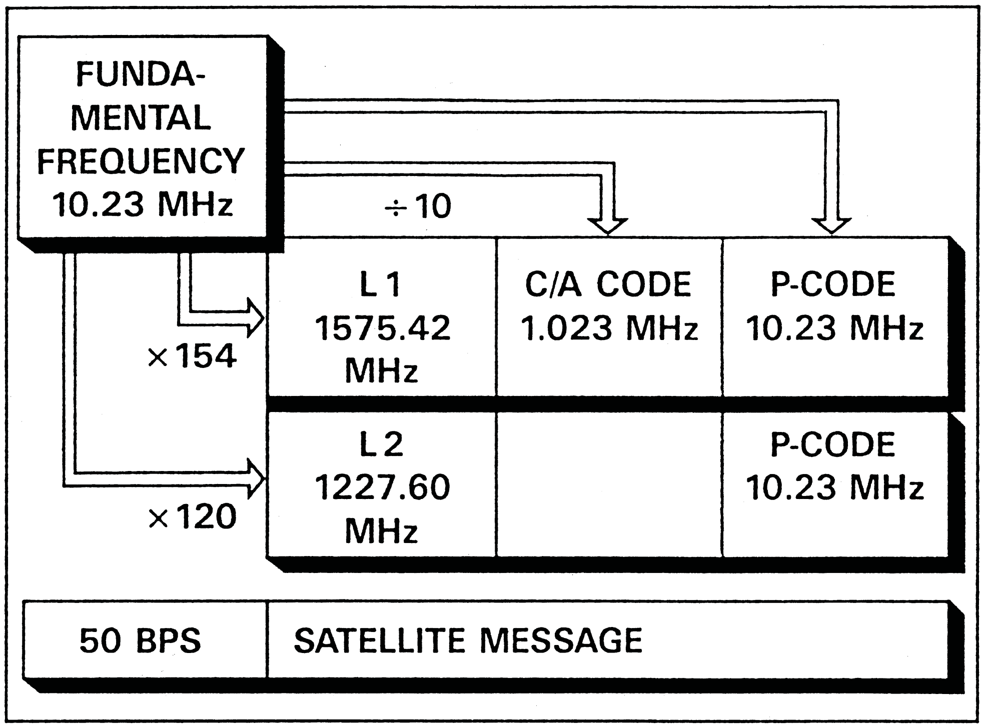
Ground Segment:

- IGS global network: reference frame for coordinates, orbits, and Earth rotation.
- Regional permanent networks (Europe, Japan, US, ...): densification of the reference frame.

2.

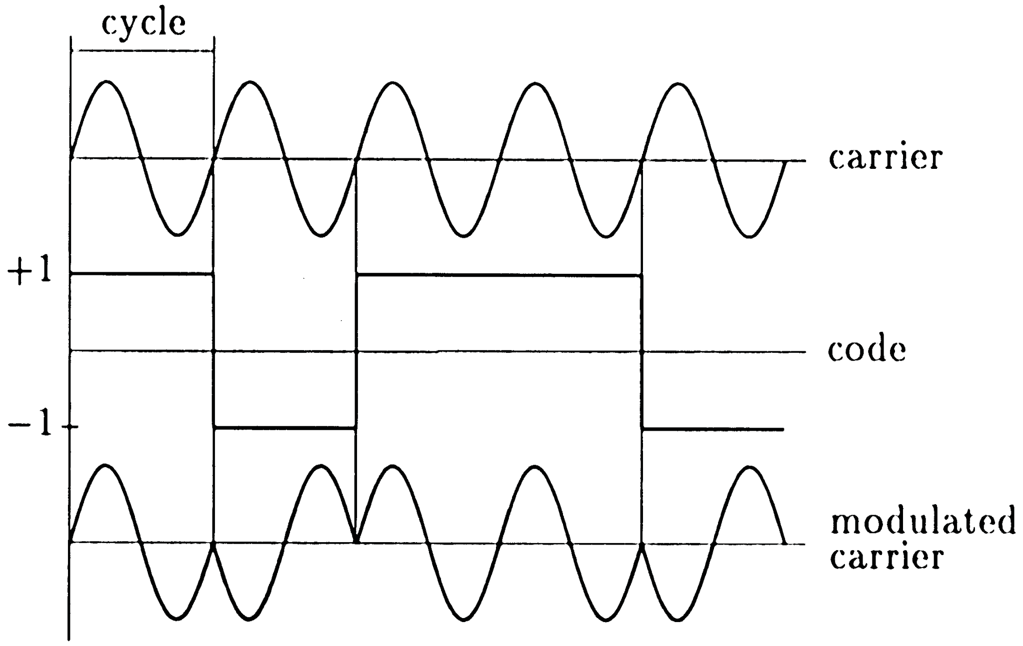
GPS/GLONASS SIGNALS

GPS SIGNALS



- Signals driven by an **atomic clock**
- Two **carrier signals** (sine waves):
  - $\boldsymbol{L}_1$: f=1575.43 MHz, λ=19 cm
  - $\boldsymbol{L}_2$: f=1227.60 MHz, λ=24 cm
- **Bits encoded** on carrier by phase modulation:
  - **C/A-code** (Clear Access / Coarse Acquisition)
  - **P-code** (Protected / Precise)
  - **Navigation Message** (Information)

GPS SIGNALS



(Hofmann-Wellenhof, 1997)

The **Navigation message** contains the following information:

- Broadcast ephemeris (orbit information)
- Satellite clock corrections
- Almanac data (approximate orbit information)
- Ionosphere information
- Satellite health status

GLONASS SIGNALS

- Two **carrier signals** with **satellite-specific frequencies**:
  - $\boldsymbol{L}_1$: f=1602 + k*0.5625 MHz
  - $\boldsymbol{L}_2$: f=1246 + k*0.4375 MHz
    with k=1,2,...,24 frequency channel number

- **Bits encoded** on carrier by phase modulation:
  - **C/A-code**: 0.511 MHz
  - **P-code**: 5.11 MHz
    Same code for all satellites.
  - **Navigation Message** (Information)

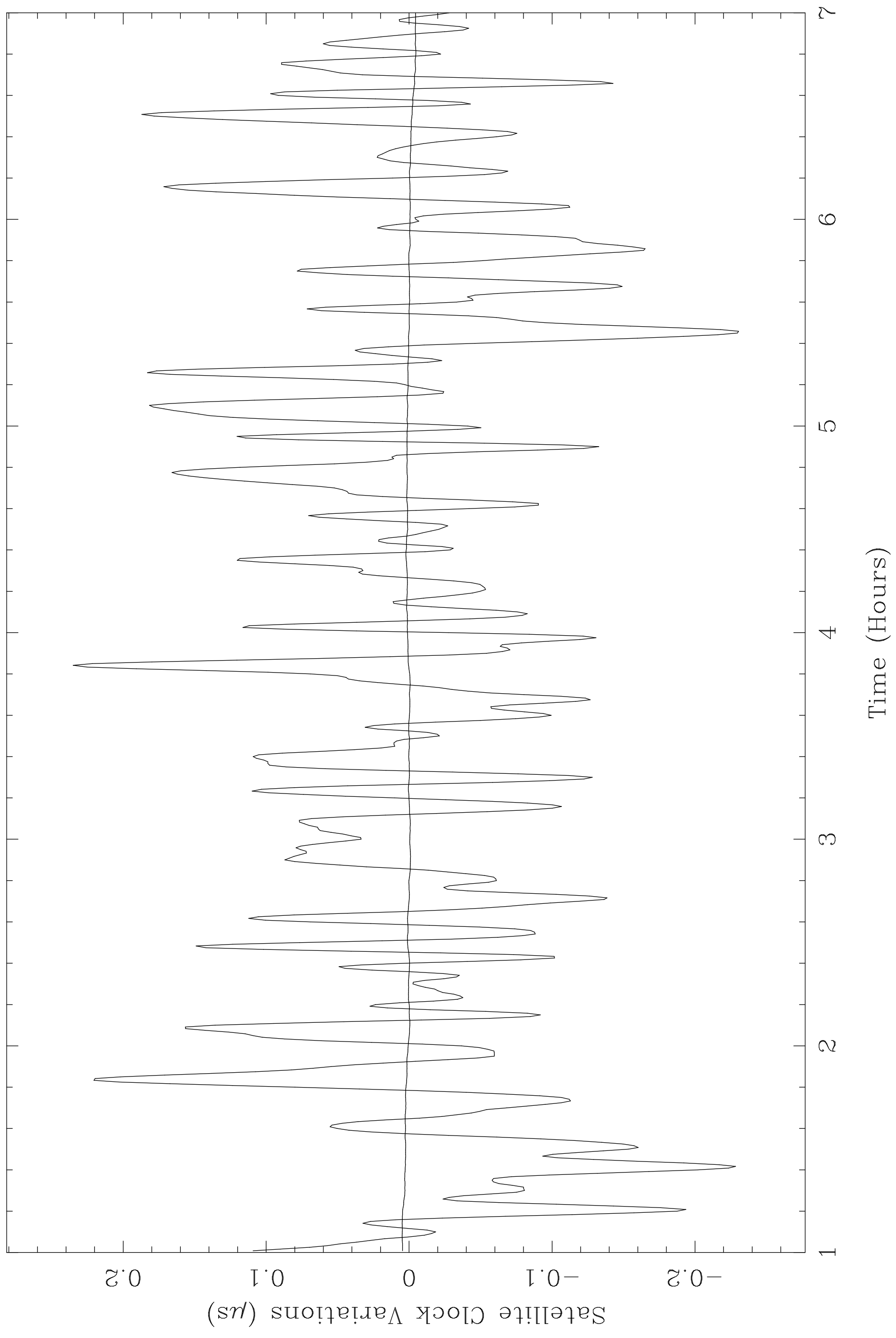
DENIAL OF ACCURACY

Two **limitations** for civilian GPS users:

- **Selective Availability (SA)**:
  - Intentional degradation of accuracy
  - **Epsilon**: navigation message contains orbit errors; apparently not used; no effect when using IGS precise orbits.
  - **Dither**: satellite clock is dithered; same effect as a satellite clock error; mitigated when using relative positioning.
- **Anti-Spoofing (AS)**:
  - Encryption of the P-code
  - Modern GPS receivers can still perform precise code measurements on L_1 and L_2
  - Increased noise level for code measurements and for L_2 carrier phase measurements.
- **GLONASS**: no degradation by **SA** or **AS**.

EXAMPLE SELECTIVE AVAILABILITY (SA)

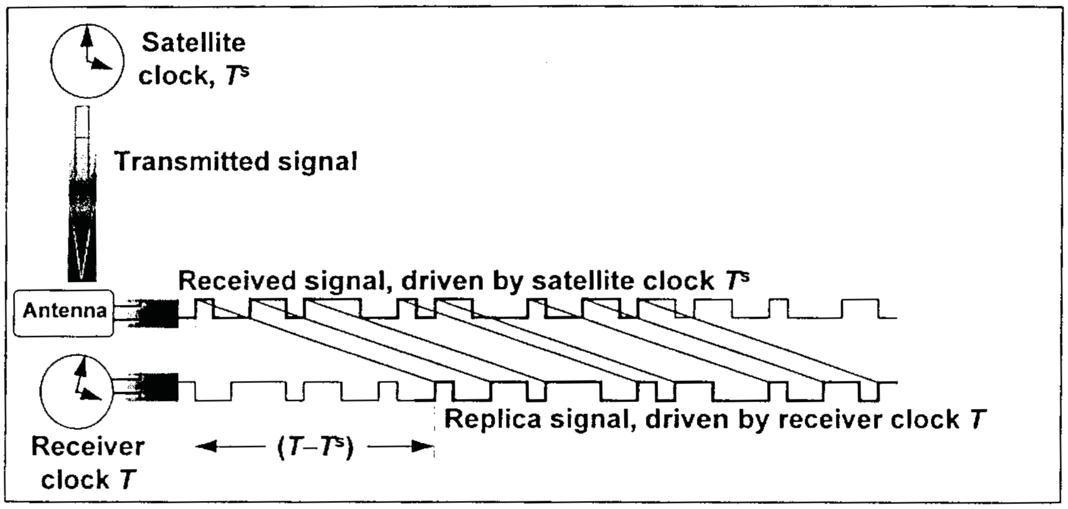
Satellite clock of PRN 15 on day 307, 1998.



3.

OBSERVATION MODEL / OBSERVATION EQUATION

PSEUDORANGE (CODE) MEASUREMENT



(Blewitt, 1996)

- The actual **pseudorange observation** P_r^s:

$$P_r^s = c\,(T_r - T^s)$$

- Clock of receiver r reads T_r when signal is received (T_r in receiver clock time).
- Clock of satellite s reads T^s when signal is emitted (T^s in satellite clock time).
- c : speed of light (in vacuum)

PSEUDORANGE (CODE) MEASUREMENT (2)

$$\begin{aligned} P_r^s &= c\,(T_r - T^s) \\ &= c\,(t_r + \delta t_r - t^s - \delta t^s) \\ &= c\,(t_r - t^s) + c\,\delta t_r - c\,\delta t^s \\ &= \rho_r^s + c\,\delta t_r - c\,\delta t^s \end{aligned}$$

where

t_r, t^s	GPS time of reception and emission
δt_r, δt^s	Receiver and satellite clock error
ρ_r^s	Range between receiver and satellite

Simplified model: atmospheric delay missing etc.

$$\rho_r^s = \sqrt{(x^s - x_r)^2 + (y^s - y_r)^2 + (z^s - z_r)^2}$$

Known from the navigation message:

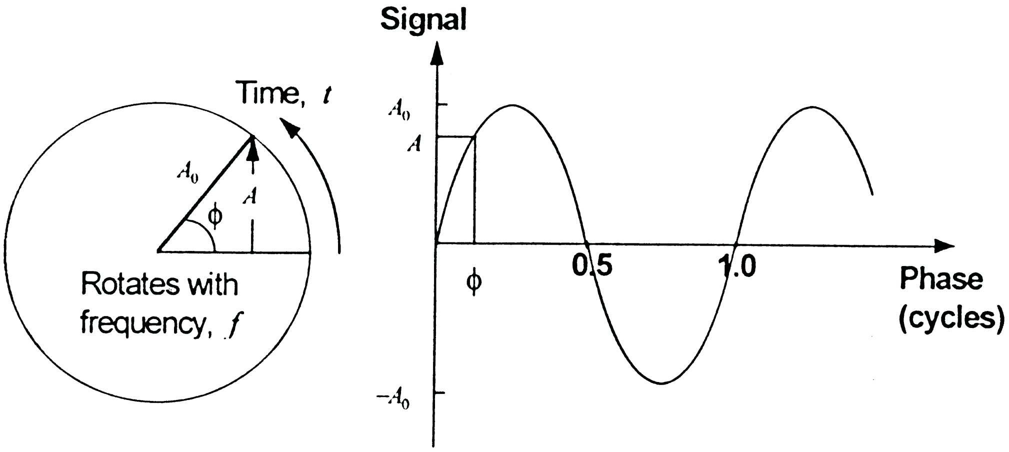
- satellite position (x^s, y^s, z^s)
- satellite clock error δt^s

4 unknows:

- receiver position (x_r, y_r, z_r)
- receiver clock error δt_r

CARRIER PHASE MEASUREMENT

A nice derivation of the carrier phase observation equation has been given by (Blewitt, 1996):



Phase Φ (in cycles) grows linearly with time t:

$$\Phi = f\,t + \Phi_0$$

with frequency f and unknown initial phase Φ_0.

Phase observation ϕ_A^j of satellite j by receiver A, when receiver clock reads T_A:

$$\phi_A^j = \Phi_{0,A} - \Phi_{GPS,A}^j$$

$\Phi_{0,A}$	Reference phase (cycles) generated by the receiver clock, at receiver clock time T_A.
$\Phi_{GPS,A}^j$	GPS signal phase (cycles) as received from satellite j at receiver clock time T_A.

CARRIER PHASE MEASUREMENT (2)

The identical GPS signal phase travels from satellite to receiver, but the receiver does not know the **integer part** of the phase:

$$\Phi^j_{GPS,A} = \Phi^{GPS,j}_A - N^j_A$$

$\Phi^{GPS,j}_A$	GPS signal phase (cycles) transmitted to receiver A at satellite clock time T^j.
N^j_A	Integer ambiguity (or initial phase bias) for satellite j and receiver A.

When the receiver looses the lock to the GPS signal phase (to count the phase cycles), a jump in N^j_A occurs, called a **cycle slip**. GPS data preprocessing therefore includes the fixing of cycle slips.

The reference phase is **commensurate** with the receiver clock, which reads T_A at time of reception:

$$\Phi_{0,A} = f_0\, T_A + \alpha_A$$

with the nominal frequency f_0 and an instrumental constant α_A.

CARRIER PHASE MEASUREMENT (3)

The GPS signal phase is commensurate with the satellite clock, which reads T^j at transmit time:

$$\Phi_A^{GPS,j} = f_0 \, T^j + \alpha^j$$

$$\Phi_{GPS,A}^j = \Phi_A^{GPS,j} - N_A^j = f_0 \, T^j + \alpha^j - N_A^j$$

α^j: instrumental constant for satellite j.

Thus the carrier phase observation equation reads:

$$\begin{aligned} \phi_A^j &= \Phi_{0,A} - \Phi_{GPS,A}^j \\ &= (f_0 \, T_A + \alpha_A) - (f_0 \, T^j + \alpha^j - N_A^j) \\ &= f_0 \, (T_A - T^j) + (N_A^j - \alpha^j + \alpha_A) \end{aligned}$$

Multiplying by the wavelength $\lambda_0 = c/f_0$ we get the phase observation in **units of length** $L_A^j = \lambda_0 \, \phi_A^j$:

$$\begin{aligned} L_A^j &= c \, (T_A - T^j) + \lambda_0 \, (N_A^j - \alpha^j + \alpha_A) \\ &= c \, (T_A - T^j) + b_A^j \end{aligned}$$

with the **pseudorange term** $c \, (T_A - T^j)$ and the **phase bias** b_A^j, and thus

$$L_A^j = \rho_A^j + c \, \delta t_r - c \, \delta t^s + b_A^j$$

IMPROVED OBSERVATION MODEL

- So far: no **atmospheric** corrections, no **relativistic** corrections
- Let us have one more look at the pseudorange observation equation

$$\begin{aligned} P_A^j &= c\,(T_A - T^j) \\ &= c\,(t_A - t^j) + c\,\delta t_A - c\,\delta t^j \end{aligned}$$

  with receive time t_A and transmit time t^j in GPS time (coordinate time).

- The term $c\,(t_A - t^j) = c\,\tau_A^j$ contains all the signal propagation (τ_A^j = light travel time):

$$c\,\tau_A^j = \left|\vec{r}_A(t_A) - \vec{r}^j(t_A - \tau_A^j)\right| + \delta\rho_{trp} + \delta\rho_{ion} + \delta\rho_{rel}$$

  where

$\vec{r}_A(t_A)$	Station position at receive time t_A
$\vec{r}^j(t^j)$	Satellite position at transmit time t^j
$\delta\rho_{trp}$	Tropospheric delay
$\delta\rho_{ion}$	Ionospheric delay
$\delta\rho_{rel}$	General relativistic corrections

- The signal propagation time $\tau_A^j = (t_A - t^j)/c$ has to be computed **iteratively**, because it appears on left and right hand side of the equation above.

IMPROVED OBSERVATION MODEL (2)

For the computation of the **geometric range** $\rho_A^j = \left|\vec{r}_A(t_A) - \vec{r}^j(t_A - \tau_A^j)\right|$ with the positions given in an **inertial frame** (e.g. J2000.0) we need:

- Satellite position $\vec{r}^j$:

  $$\vec{r}^j = \vec{r}_0^j + \delta\vec{r}_{ant}^j$$

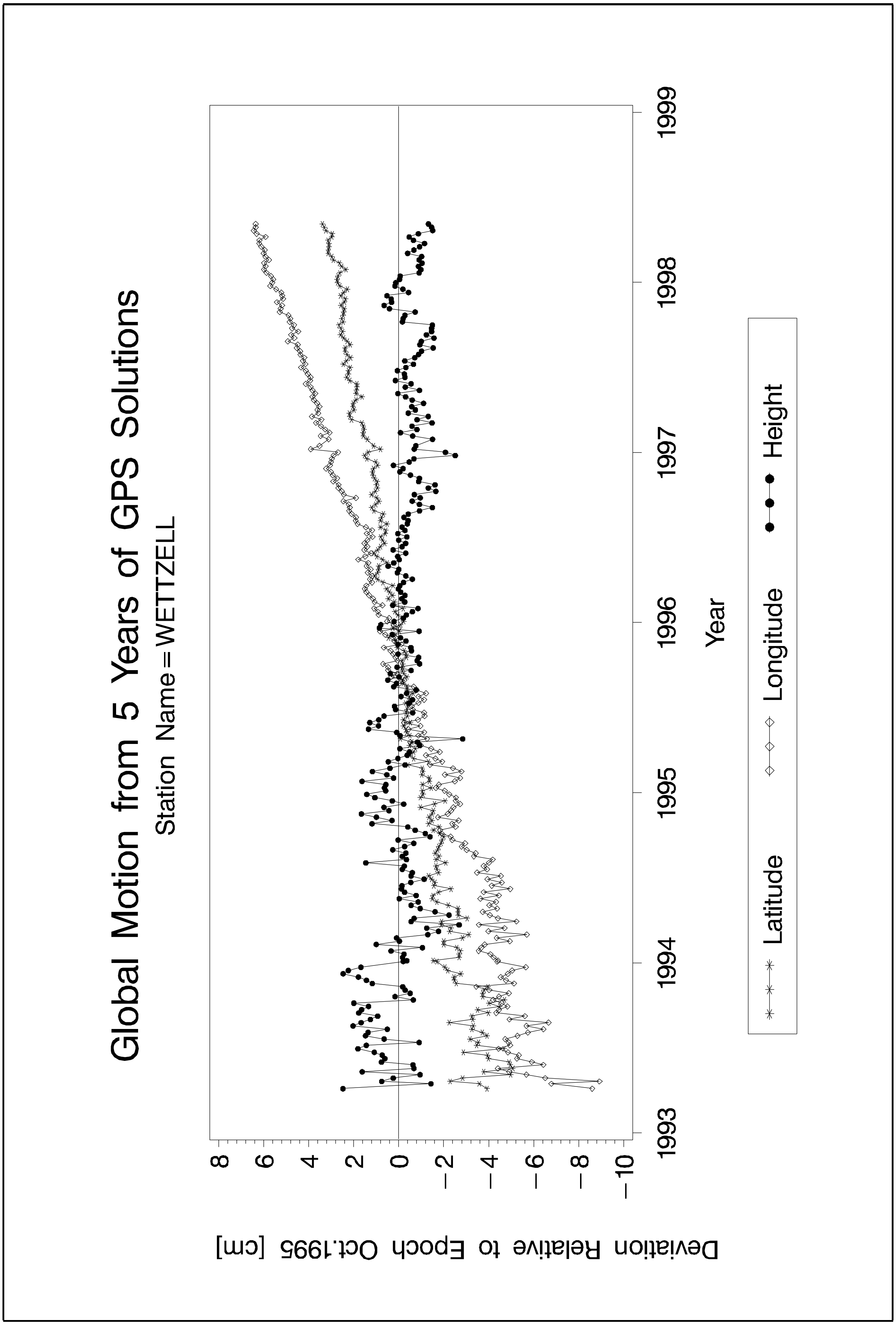
  $\delta\vec{r}_{ant}^j$: difference between **antenna phase center** $\vec{r}^j$ and **center of mass** $\vec{r}_0^j$ of the satellite.

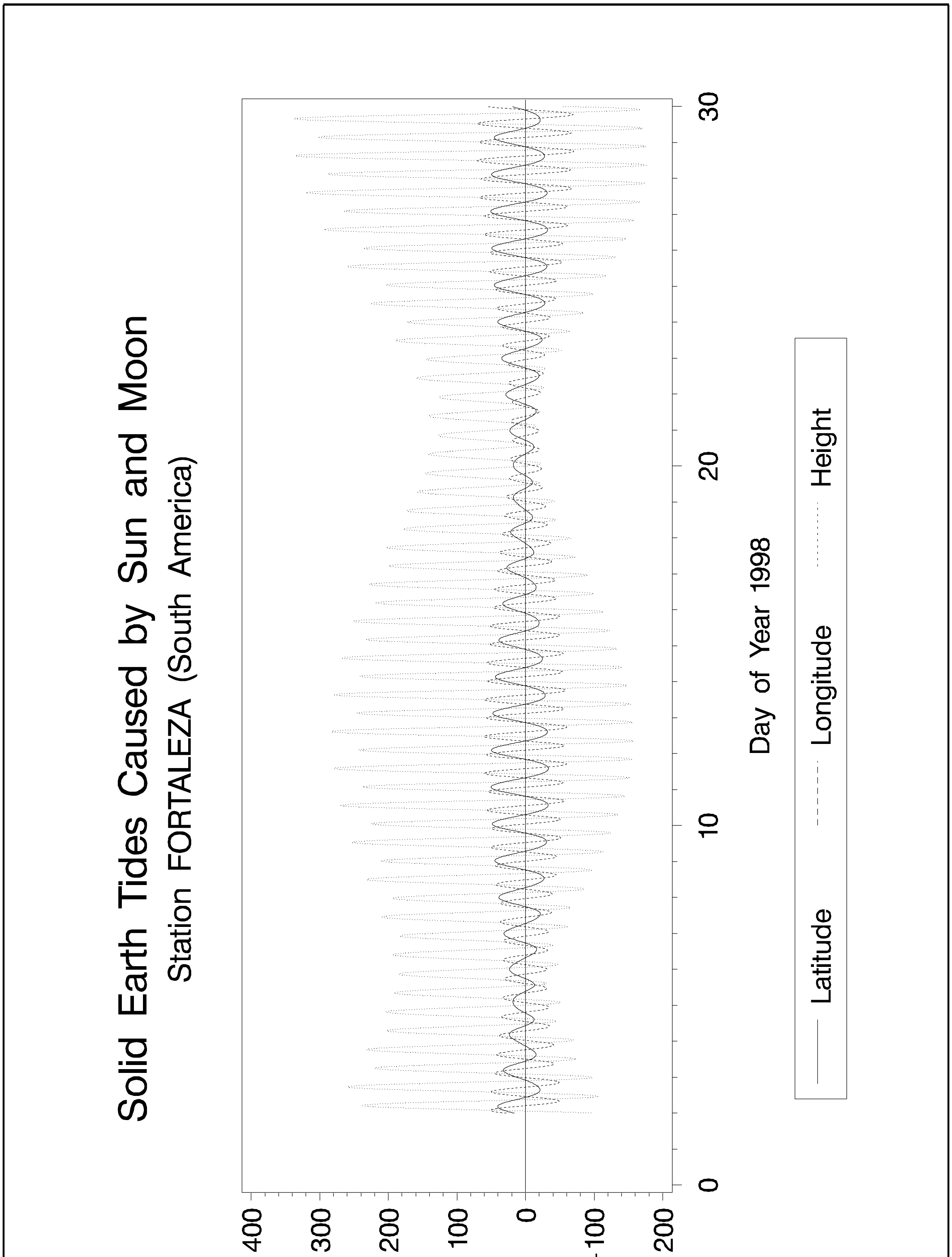
- Receiver position $\vec{r}_A$:

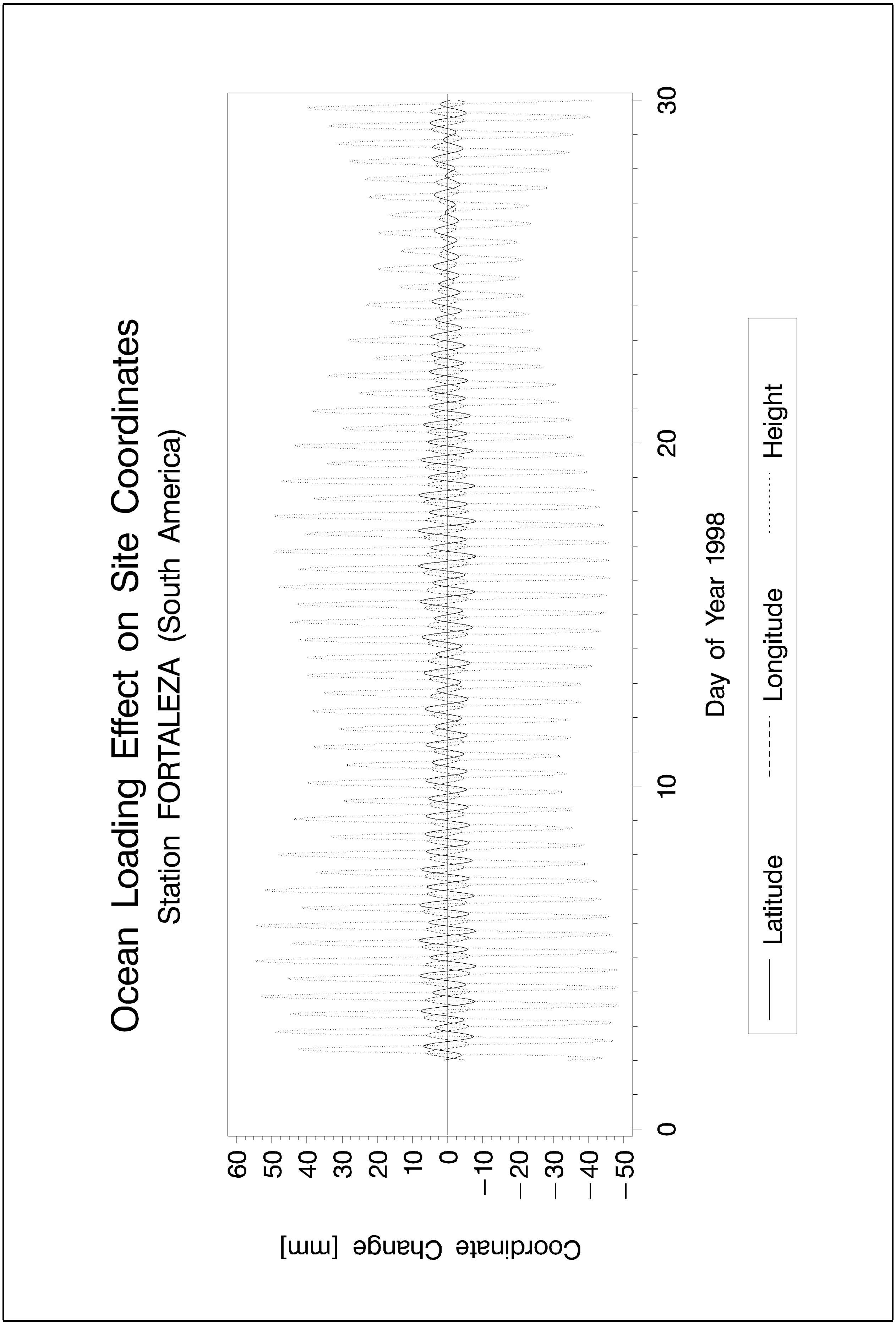
  $$\vec{r}_A = P\,N\,U\,X\,Y\,\vec{r}_{A,earth}$$

  $$\begin{aligned}\vec{r}_{A,earth} \;=\; & \vec{r}_{A,0} + \vec{v}_A\,(t - t_0) + \delta\vec{r}_{A,solid} + \delta\vec{r}_{A,pole} \\ & + \delta\vec{r}_{A,ocean} + \delta\vec{r}_{A,atmos} + \delta\vec{r}_{A,ant}\end{aligned}$$

P, N, U, X, Y	Transformation matrices (Earth-fixed – inertial)
$\delta\vec{r}_{A,ocean}$, $\delta\vec{r}_{A,atmos}$	Ocean and atmos. loading
$\delta\vec{r}_{A,solid}$, $\delta\vec{r}_{A,pole}$	Solid Earth and pole tides
$\delta\vec{r}_{A,ant}$	Antenna phase center variation


Global Motion from 5 Years of GPS Solutions
Station Name=WETTZELL
Deviation Relative to Epoch Oct.1995 [cm]
8 6 4 2 0 −2 −4 −6 −8 −10
1993 1994 1995 1996 1997 1998 1999
Year
Latitude
Longitude
Height


Solid Earth Tides Caused by Sun and Moon
Station FORTALEZA (South America)
Coordinate Change [mm]
400
300
200
100
0
−100
−200
0
10
20
30
Day of Year 1998
Latitude
Longitude
Height


Ocean Loading Effect on Site Coordinates
Station FORTALEZA (South America)
Coordinate Change [mm]
60 50 40 30 20 10 0 −10 −20 −30 −40 −50
0 10 20 30
Day of Year 1998
Latitude
Longitude
Height

4.

DIFFERENCING, LINEAR COMBINATIONS, AND AMBIGUITY RESOLUTION

DIFFERENCING: SINGLE DIFFERENCES

Idea: Elimination of **satellite clock error** by forming differences between **two receivers** A and B (baseline $A - B$) observing the same satellite j.

$$L_A^j = \rho_A^j + \delta\rho_A^j + c\,\delta t_A - c\,\delta t^j + b_A^j$$
$$L_B^j = \rho_B^j + \delta\rho_B^j + c\,\delta t_B - c\,\delta t^j + b_B^j$$

$$\begin{aligned}
\Delta L_{AB}^j &= L_A^j - L_B^j \\
&= (\rho_A^j + \delta\rho_A^j + c\,\delta t_A - c\,\delta t^j + b_A^j) - \\
&\quad\ (\rho_B^j + \delta\rho_B^j + c\,\delta t_B - c\,\delta t^j + b_B^j) \\
&= \Delta\rho_{AB}^j + \Delta\delta\rho_{AB}^j + c\,\Delta\delta t_{AB} + \Delta b_{AB}^j
\end{aligned}$$

- Satellite clock error not fully eliminated, because

  $$\delta t^j(t_A - \tau_A^j) \neq \delta t^j(t_B - \tau_B^j) = \delta t^j(t_A - \Delta\delta t_{AB} - \tau_B^j)$$

- The receivers have to measure simultaneously: $\Delta\delta t_{AB} < 1$ ms (neglection smaller than 1 mm in range assuming **satellite clock changes** under SA not larger than 60 m within 1 minute).
- For long baselines the light travel time τ_A^j and τ_B^j are different.

DIFFERENCING: DOUBLE DIFFERENCES

Idea: Elimination of **receiver clock error** by forming differences between **two satellites** j and k observed from A and B.

$$\Delta L^j_{AB} = \Delta\rho^j_{AB} + \Delta\delta\rho^j_{AB} + c\,\Delta\delta t_{AB} + \Delta b^j_{AB}$$
$$\Delta L^k_{AB} = \Delta\rho^k_{AB} + \Delta\delta\rho^k_{AB} + c\,\Delta\delta t_{AB} + \Delta b^k_{AB}$$

$$\begin{aligned}\nabla\Delta L^{jk}_{AB} &= \Delta L^j_{AB} - \Delta L^k_{AB}\\ &= (\Delta\rho^j_{AB} + \Delta\delta\rho^j_{AB} + c\,\Delta\delta t_{AB} + \Delta b^j_{AB}) -\\ &\quad (\Delta\rho^k_{AB} + \Delta\delta\rho^k_{AB} + c\,\Delta\delta t_{AB} + \Delta b^k_{AB})\\ &= \nabla\Delta\rho^{jk}_{AB} + \nabla\Delta\delta\rho^{jk}_{AB} + \nabla\Delta b^{jk}_{AB}\end{aligned}$$

- Although the receiver clock error is eliminated we still need to know it to better than 0.3 μs to compute the **satellite positions** at the correct epoch **in GPS time** (satellite positions — broadcast or IGS – refer to GPS time).
- Assuming a satellite velocity of about 3 km/s relative to the station the satellite moves about 1 mm in 0.3 μs.
- Double difference ambiguity is integer $\nabla\Delta b^{jk}_{AB} = \lambda \cdot \nabla\Delta N^{jk}_{AB}$ (instrumental biases are cancelling).

DIFFERENCING: TRIPLE DIFFERENCES

Idea: Elimination of **ambiguities** by forming differences between **consecutive epochs** i and $i+1$.

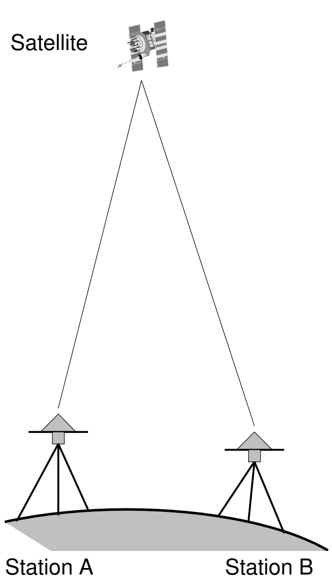
$$
\begin{aligned}
\nabla\Delta L_{AB}^{jk}(i) &= \nabla\Delta\rho_{AB}^{jk}(i) + \nabla\Delta\delta\rho_{AB}^{jk}(i) \\
&\quad +\lambda \cdot \nabla\Delta N_{AB}^{jk} \\
\nabla\Delta L_{AB}^{jk}(i+1) &= \nabla\Delta\rho_{AB}^{jk}(i+1) + \nabla\Delta\delta\rho_{AB}^{jk}(i+1) \\
&\quad +\lambda \cdot \nabla\Delta N_{AB}^{jk}
\end{aligned}
$$

Difference between epochs i and $i+1$, denoted by $(i, i+1)$:

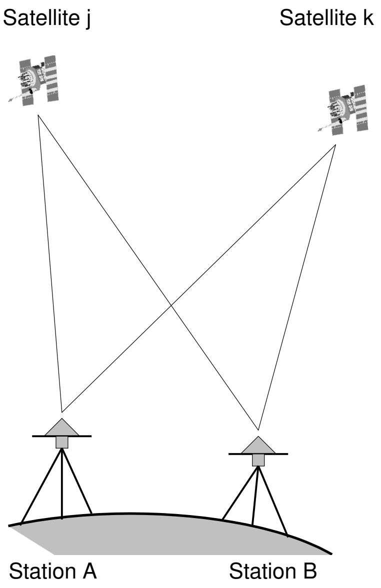
$$
\begin{aligned}
\nabla\Delta L_{AB}^{jk}(i,i+1) &= \nabla\Delta L_{AB}^{jk}(i) - \nabla\Delta L_{AB}^{jk}(i+1) \\
&= \nabla\Delta\rho_{AB}^{jk}(i,i+1) \\
&\quad +\nabla\Delta\delta\rho_{AB}^{jk}(i,i+1)
\end{aligned}
$$

- **Cycles slips** show up as **outliers** in triple differences and can be removed.
- A triple difference often serves as a **first, robust solution** for the relative coordinates between stations A and B.

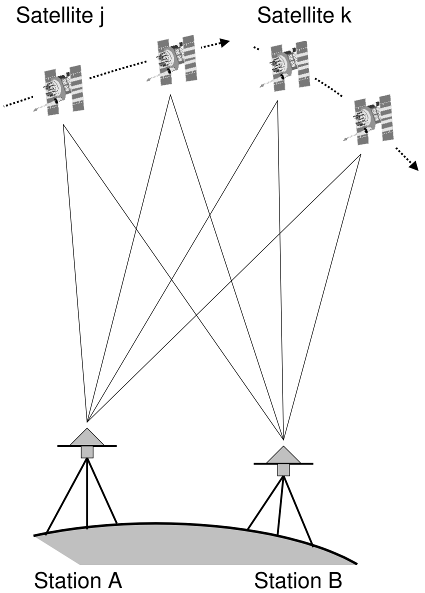
DIFFERENCING: ILLUSTRATIONS



Single Differences



Double Differences



Triple Differences

LINEAR COMBINATIONS

Four observation types available:

- L_1: Carrier phase on L_1, frequency f_1
- L_2: Carrier phase on L_2, frequency f_2
- P_1: Pseudorange on L_1 channel (C/A or P code)
- P_2: Pseudorange on L_2 channel (P code)

Simplified double difference observation equations:

$$\begin{aligned} L_1 &= \rho + I_1 + \lambda_1 N_1 \\ L_2 &= \rho + I_2 + \lambda_2 N_2 \\ P_1 &= \rho - I_1 \\ P_2 &= \rho - I_2 \end{aligned}$$

where:

ρ	Non-dispersive delay, contains **geometry, clocks, troposphere**.
I_1, I_2	Dispersive delay on L_1 and L_2 caused by the **ionosphere**. Note the opposite sign of the ionospheric delay for **phase** versus **group** (code) delays.
N_1, N_2	Integer **ambiguities** on L_1 and L_2

LINEAR COMBINATIONS (2)

The ionospheric delay I proportional to f^{-2}:

$$I_1 = k/f_1^2, \qquad I_2 = k/f_2^2$$

This means that

$$I_2 = \frac{f_1^2}{f_2^2}\, I_1$$

We thus have:

$$\begin{aligned} L_1 &= \rho + I_1 + \lambda_1 N_1 \\ L_2 &= \rho + \frac{f_1^2}{f_2^2}\, I_1 + \lambda_2 N_2 \\ P_1 &= \rho - I_1 \\ P_2 &= \rho - \frac{f_1^2}{f_2^2}\, I_1 \end{aligned}$$

- **4 equations** and **4 unknowns**, namely ρ, I_1, N_1, and N_2.
- Sufficient information to obtain the geometry ρ and the delay I_1, and resolve the ambiguities N_1 and N_2, if the data were precise enough.
- Form linear combinations (LCs) of the four equations to **eliminate error sources** (e.g. ionosphere) or to help in **cycle slip fixing** and **ambiguity resolution**.

LINEAR COMBINATIONS (3)

Geometry-free LC:

$$\begin{aligned} L_I &= L_1 - L_2 \\ &= (\rho + I_1 + \lambda_1 N_1) - (\rho + \frac{f_1^2}{f_2^2} I_1 + \lambda_2 N_2) \\ &= (1 - \frac{f_1^2}{f_2^2})\, I + (\lambda_1 N_1 - \lambda_2 N_2) \end{aligned}$$

$$\begin{aligned} P_I &= P_1 - P_2 \\ &= -(1 - \frac{f_1^2}{f_2^2})\, I \end{aligned}$$

Purpose: Information about the **ionosphere**; estimation of ionosphere models; cycle slip fixing.

LINEAR COMBINATIONS (4)

Widelane LCs:

$$
\begin{aligned}
L_w &= \lambda_w(\Phi_1 - \Phi_2), \qquad\qquad \lambda_w = 86cm \\
&= \frac{f_1}{f_1 - f_2} L_1 - \frac{f_2}{f_1 - f_2} L_2 \\
&= \rho + \frac{f_1}{f_2}\, I_1 + \lambda_w(N_1 - N_2)
\end{aligned}
$$

$$
\begin{aligned}
P_w &= \frac{f_1}{f_1 + f_2} P_1 + \frac{f_2}{f_1 + f_2} P_2 \\
&= \rho + \frac{f_1}{f_2}\, I_1
\end{aligned}
$$

Purpose: Widelane ambiguity resolution $N_w = N_1 - N_2$ and cycle slip fixing, advantage of the long wavelength λ_w.

Melbourne-Wübbena LC:

$$MW = L_w - P_w = \lambda_w(N_1 - N_2)$$

Purpose: Widelane ambiguity resolution independent of geometry and ionosphere. The large noise of pseudorange observations is problematic.

LINEAR COMBINATIONS (5)

Ionosphere-free LC:

$$\kappa_1 = \frac{f_1^2}{f_1^2 - f_2^2}, \quad \kappa_2 = -\frac{f_2^2}{f_1^2 - f_2^2}$$

We then form:

$$\begin{aligned} L_c &= \kappa_1 L_1 + \kappa_2 L_2 \\ &= \kappa_1(\rho + I_1 + \lambda_1 N_1) + \kappa_2(\rho + \frac{f_1^2}{f_2^2} I_1 + \lambda_2 N_2) \\ &= \rho + (\kappa_1 \lambda_1 N_1 + \kappa_2 \lambda_2 N_2) \end{aligned}$$

$$\begin{aligned} P_c &= \kappa_1 P_1 + \kappa_2 P_2 \\ &= \rho \end{aligned}$$

Purpose: Eliminate ionosphere from observations; main observable for baselines longer than 5–10 km. Disadvantage: noise 3 times higher than for L_1.

If we introduce $N_2 = N_1 - N_w$ (assuming that the widelane ambiguity N_w is known) into L_c we get the so-called narrowlane:

$$L_c = \rho + (\kappa_1 \lambda_1 + \kappa_2 \lambda_2) N_1 - \kappa_2 \lambda_2 N_w$$

$$\lambda_c = \kappa_1 \lambda_1 + \kappa_2 \lambda_2 = 10.7cm$$

Ambiguity Resolution

- **Goal**: resolve phase ambiguities $\nabla\Delta N_{AB}^{jk}$ to the correct integer numbers (ambiguity fixing).
- Only possible on the **double difference level** because of instrumental biases.

Gain in accuracy:

- Short observation times (few minutes):
  - Huge gain in accuracy, typically improvement from 1 m (free) to 1 cm (fixed).
- Long sessions (e.g. 1 day):
  - Gain in accuracy much smaller, typically a factor of 2 in east–west component of site coordinates.

Resolution strategies:

- Short baselines: search algorithms searching for best set of integer ambiguities.
- Long baselines: usually procedure in two steps:
  - First resolve $N_w = N_1 - N_2$ using widelane L_w or Melbourne-Wübbena MW LC
  - Resolve N_1 with narrowlane LC (N_w known)

5.

IMPORTANT ERROR SOURCES

- Satellite orbits
- Tropospheric refraction
- Ionospheric refraction
- Antenna phase center variations
- Multipath

GPS/GLONASS ORBIT TYPES AVAILABLE TODAY

- Several **different orbit sources** are available today to the geodetic and geophysical GPS/GLONASS community.
- They vary in **quality** and in the **time delay** with which they are made available:

Orbit Type	Quality (m)	Delay of Availability
Broadcast Orbit	3.0 m	Real Time
DMA Precise Orbit	3.0 m	After 4-8 Weeks
IGS Predicted Orbit	.80 m	Real Time
IGS Rapid Orbit	.06 m	After 16 Hours
IGS Final Orbit	.04 m	After 11 Days
GLONASS Orbits	.25 m	After 4 weeks

IMPACT OF ORBIT ERRORS ON ESTIMATED SITE COORDINATES (1)

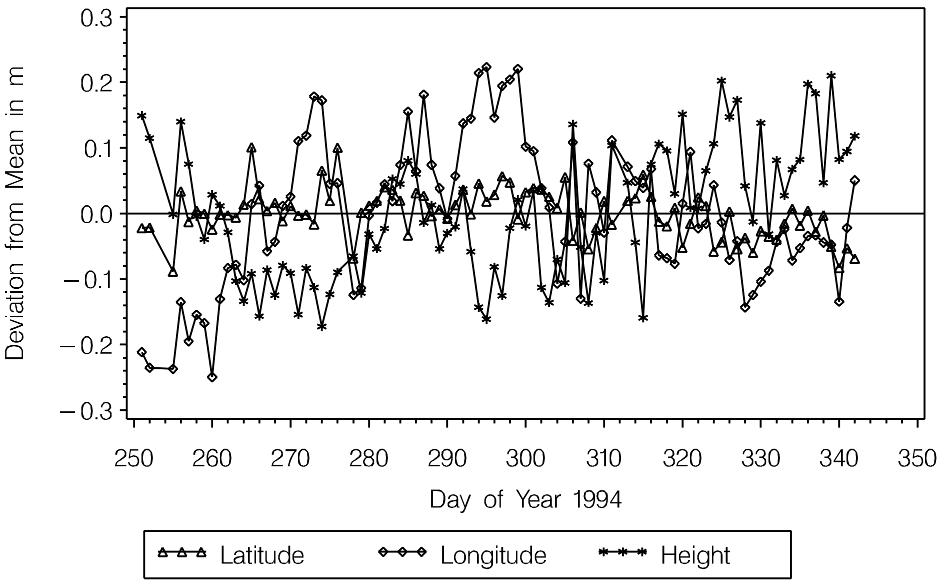
- **(Bauersima, 1983)** computes the error Δr in a component of a baseline of length l as a function of an orbit error of size ΔR:
$$\Delta r = \frac{l}{\rho} \cdot \Delta R \approx \frac{l(km)}{25'000(km)} \cdot \Delta R$$
ρ is the mean distance between the survey area and the satellite system.

- Impact of orbit errors on the estimated station coordinates for different baseline lengths:

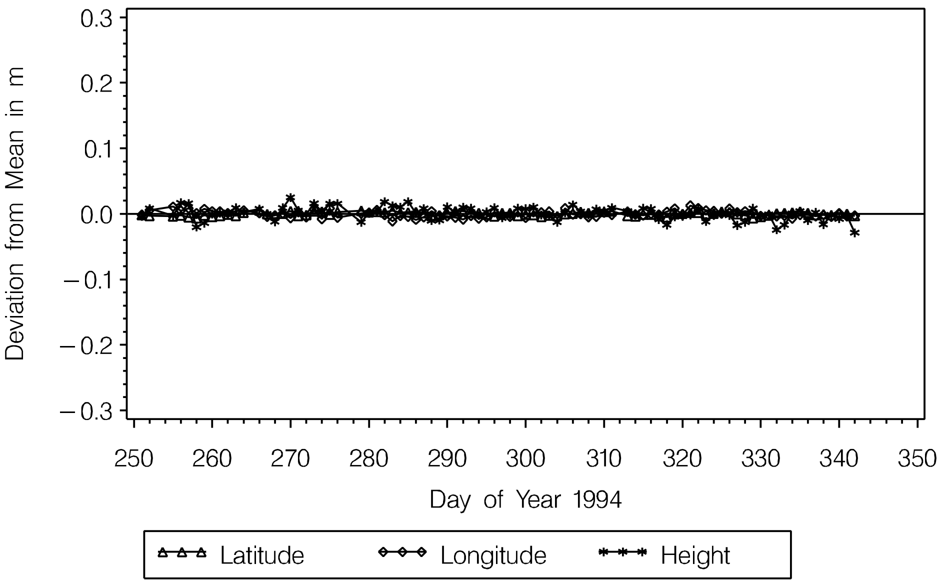
Orbit Error	Baseline Length	Baseline Error in ppm	Baseline Error in mm
25 m	1 km	1 ppm	1 mm
25 m	10 km	1 ppm	10 mm
25 m	100 km	1 ppm	100 mm
25 m	1000 km	1 ppm	1000 mm
2.5 m	10 km	.1 ppm	1 mm
2.5 m	100 km	.1 ppm	10 mm
2.5 m	1000 km	.1 ppm	100 mm
.25 m	100 km	.01 ppm	1 mm
.25 m	1000 km	.01 ppm	10 mm
.05 m	100 km	.002 ppm	- mm
.05 m	1000 km	.002 ppm	.5 mm

IMPACT OF ORBIT ERRORS ON ESTIMATED SITE COORDINATES (2)

Daily Repeatabilities of Latitude, Longitude, Height of the Baseline Onsala – Graz (from 8.9.94 – 8.12.94) Using Broadcast Orbits



Daily Repeatabilities of Latitude, Longitude, Height of the Baseline Onsala – Graz (from 8.9.94 – 8.12.94) Using IGS Orbits



ATMOSPHERIC REFRACTION

Two regions of the atmosphere important for GPS:

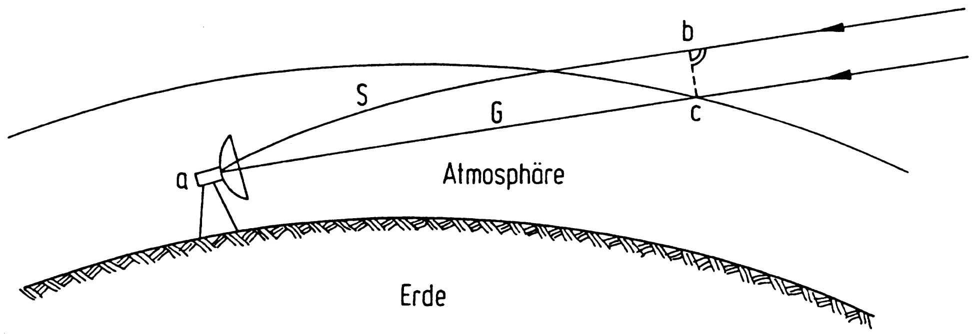
- The **troposphere**, **non-dispersive** for GPS signals, extending to a height of about 10 km.
- The **ionosphere**, **dispersive**, extending from about 50–1000 km.

The electrical path length L of a signal propagating along path S through the atmosphere with the refractive index n is defined as

$$L = \int_S n\,ds$$

$$\Delta L = \int_S n\,ds - G, \quad \Delta L = \int_S (n-1)ds + S - G$$

S is the signal path, G the geometric path, ΔL the delay. $S - G$ is about 3 cm at 10^o elevation (bending).



(Elgered, 1993)

TROPOSPHERIC REFRACTION

- Delay is **frequency-independent** (same for L_1, L_2)
- Total delay is about **2.3 m** (ca. 8 nsec)
- Two parts:
  - **Dry delay** (hydrostatic delay):
    - ▷ 90% of the total delay
    - ▷ mainly depending on the surface pressure and therefore easy to model
    - ▷ slow temporal variations (1cm/6h)
  - **Wet delay**:
    - ▷ maximal value of about 40 cm
    - ▷ rapidly changing in time and space
    - ▷ difficult to model or to predict (unknown distribution of water vapor).
- Delay usually given by a formula of the type:

$$\delta\rho_{trp}(z) = f_{dry}(z) \cdot \delta\rho_{dry}(0) + f_{wet}(z) \cdot \delta\rho_{wet}(0)$$

$f_{dry}(z)$, $f_{wet}(z)$	**Mapping functions** (e.g. $1/cos(z)$)
$\delta\rho_{dry}(0)$, $\delta\rho_{wet}(0)$	**Zenith delays** ($z = 0$)
z	Zenith angle

IMPACT OF TROPOSPHERE ON COORDINATES

- Troposphere is the **most important error source** since orbit errors are well under control with the IGS products.
- **Relative troposphere error** (biases in the tropospheric delays of one site relative to the other site of a baseline):
  - Main effect in station height difference between the two stations
  - **1 cm** error in tropospheric zenith delay $\rightarrow$ **3 cm** error in height (factor 3).
- **Absolute troposphere error** (same bias at both ends of a baseline):
  - has a smaller effect than relative error
  - **10 cm** error in absolute zenith delays $\rightarrow$ scale factor of **0.05 ppm** in the baseline.

Two possibilities to reduce troposphere errors:

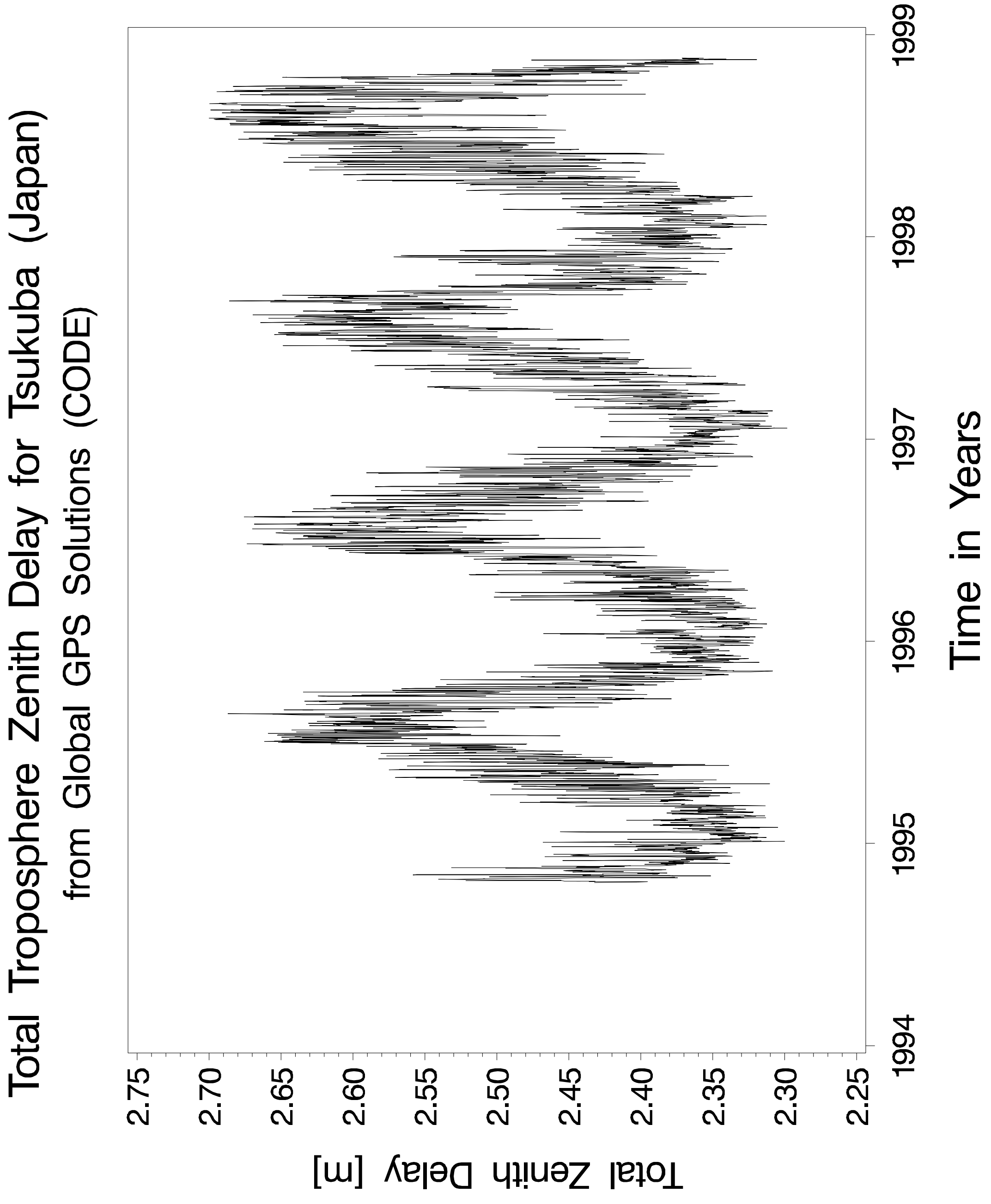
1. **Modeling** tropospheric refraction **without** using GPS observations.
2. Use of GPS observations to **estimate** tropospheric parameters.

MODELING THE TROPOSPHERE

- The common models (Saastamoinen,Hopfield, Essen-Froome, ...) compute the delay using values for **pressure, temperature, and humidity** at the site. Two main possibilities to obtain such meteorological values:
  - **Extrapolation** of standard values from sea level to the height of the site (standard atmosphere)
  - Meteorological **measurements** at the sites
- **Problems:**
  - A standard atmosphere does not reflect the **real** atmospheric conditions (daily and seasonal changes, geographical location, ...)
  - Surface measurements **not accurate enough** (e.g. error in temperature of 1 deg $\rightarrow$ 2.5 cm error in zenith delay).
  - Surface values are **not representative** for the entire atmosphere above the site.
- **Water vapor radiometers** (WVR) measurements as an alternative, but very expensive.

ESTIMATING TROPOSPHERE PARAMETERS

- The most common procedure is the estimation of **site-specific troposphere zenith delays**, either:
  - estimation as **piece-wise linear** functions or as **a constant** for each interval with least squares methods, typically with a time resolution of 1-2 hours.
  - estimation as a **stochastic process** (random walk) using Kalman filter techniques.
- When processing **small networks** ($<$ 100 km) in a differential mode (double differences), troposphere zenith delays cannot be estimated in an absolute sense, only relative to a reference site, because the parameters are **highly correlated** (almost the same zenith angles to the satellite).
- Comparison with WVR measurements: troposphere zenith delay estimates from GPS have a quality of about **3–8 mm**.
- There are quite a few projects to use GPS to obtain water vapor information for **meteorology** and **climatology**.


Total Troposphere Zenith Delay for Tsukuba (Japan)
from Global GPS Solutions (CODE)
Total Zenith Delay [m]
2.75
2.70
2.65
2.60
2.55
2.50
2.45
2.40
2.35
2.30
2.25
1994
1995
1996
1997
1998
1999
Time in Years

IONOSPHERIC REFRACTION

- The GPS signal is delayed in the ionosphere due to the interaction with the **free electrons**.
- The number of free electrons depends on the **activity of the Sun** (e.g the **11-year solar cycle**, next maximum around beginning of year 2001), the time of the day, the latitude, ...
- The delay is (approximately) proportional to the frequency f^{-2}:

$$\delta\rho_{ion} = \pm\, a \cdot \frac{E}{f^2} \cdot \frac{1}{\cos z'}$$

E	Total number of free electrons in a column with a cross-section of $1m^2$ (total electron content TEC in TECU: 10^{16} free electrons/m^2)
a	Constant: $a = 40.3 \cdot 10^{16}\, m\, s^{-2}\, TECU^{-1}$
z'	Zenith angle at the height of the ionosphere

- The positive sign is valid for **pseudoranges** (group delay), the negative sign for **phases** (phase delay).
- The total vertical delay may vary between about **1-20 m**.

IMPACT OF IONOSPHERE ON COORDINATES

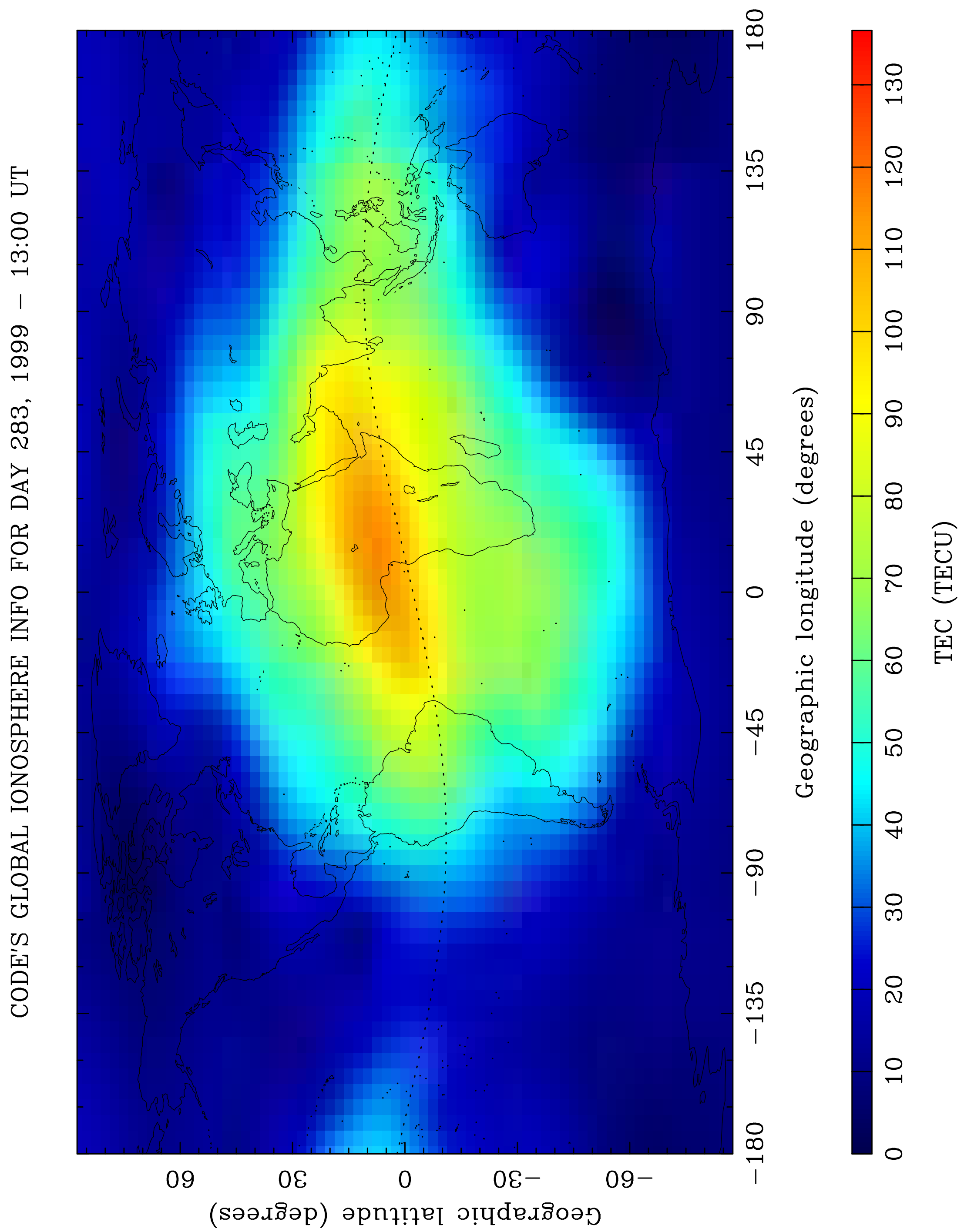
- The ionospheric refraction is the **most important error source** if L_1, L_2, or the widelane L_w are processed.
- Main effect on the station coordinates is a **scaling of the baselines** (if ionospheric delays are not modeled, the baselines are **too short**).
- An estimate of the size of the **scale factor** to be expected for a baseline with length l is given by:

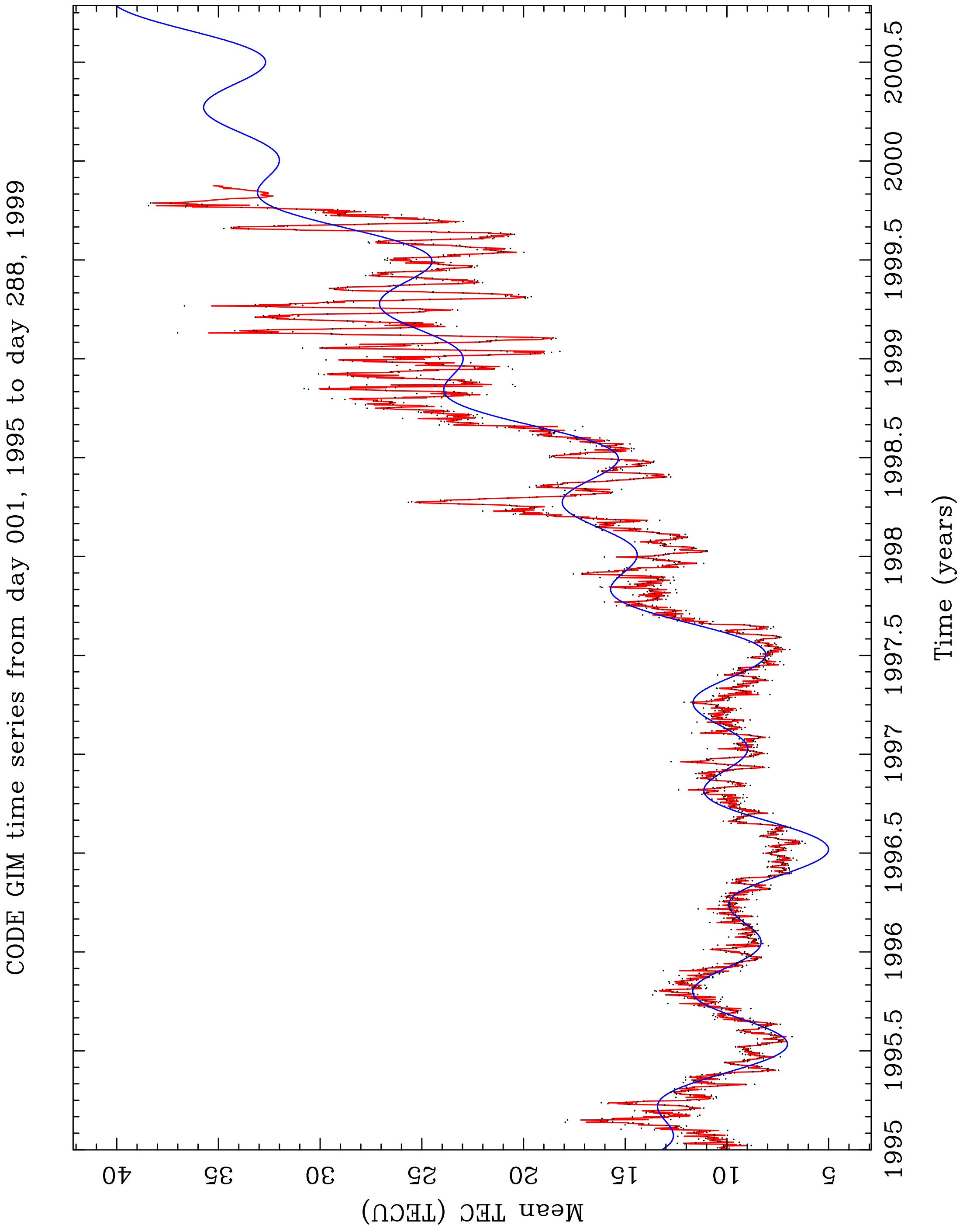
$$\frac{dl}{l} \approx -0.08 \cdot E \quad [ppm] \quad (E\ in\ TECU)$$

- **Example**: Network of 4 by 4 km in Switzerland in 1989: $E \approx 50$ TECU, $dl/l \approx -4$ ppm, and $dl \approx -1.6$ cm for the baseline error.

IONOSPHERIC MODELING AND ESTIMATION

- **External information**:
  - A very simple model to correct for the ionospheric delays is part of the GPS navigation message.
  - **Regional and/or global ionosphere models** are routinely computed by several analysis centers of the **IGS** and made available on the internet.
  - Such models help to eliminate the **scale factor** in the GPS results and to **resolve ambiguities** on long baselines (and long sessions).
- **Estimation of ionosphere parameters**:
  - **Ionosphere models** are estimated in general by assuming that all free electrons are concentrated in an **infinitesimal layer** (single layer model) at a height of typically **450 km**. The electron density in this layer is then approximated by polynomials, spherical harmonics functions, or triangles.
  - The **geometry-free linear combination** L_I of L_1 and L_2 and/or P_1 and P_2 is used as observable in the estimation process.


CODE'S GLOBAL IONOSPHERE INFO FOR DAY 283, 1999 – 13:00 UT
Geographic longitude (degrees)
Geographic latitude (degrees)
TEC (TECU)
−180
−135
−90
−45
0
45
90
135
180
60
30
0
−30
−60
0
10
20
30
40
50
60
70
80
90
100
110
120
130

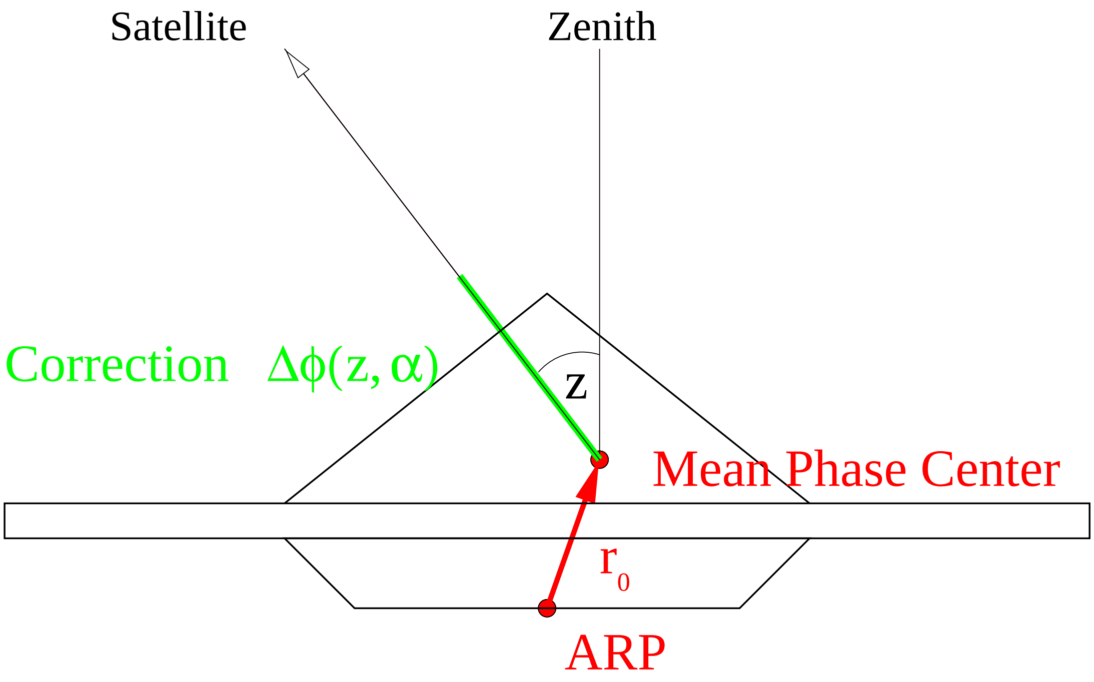

CODE GIM time series from day 001, 1995 to day 288, 1999
Mean TEC (TECU)
Time (years)
5
10
15
20
25
30
35
40
1995
1995.5
1996
1996.5
1997
1997.5
1998
1998.5
1999
1999.5
2000
2000.5

ANTENNA PHASE CENTER VARIATIONS

Antenna phase center variations $\delta\vec{r}_{A,ant}$ are best described by the following **model**:

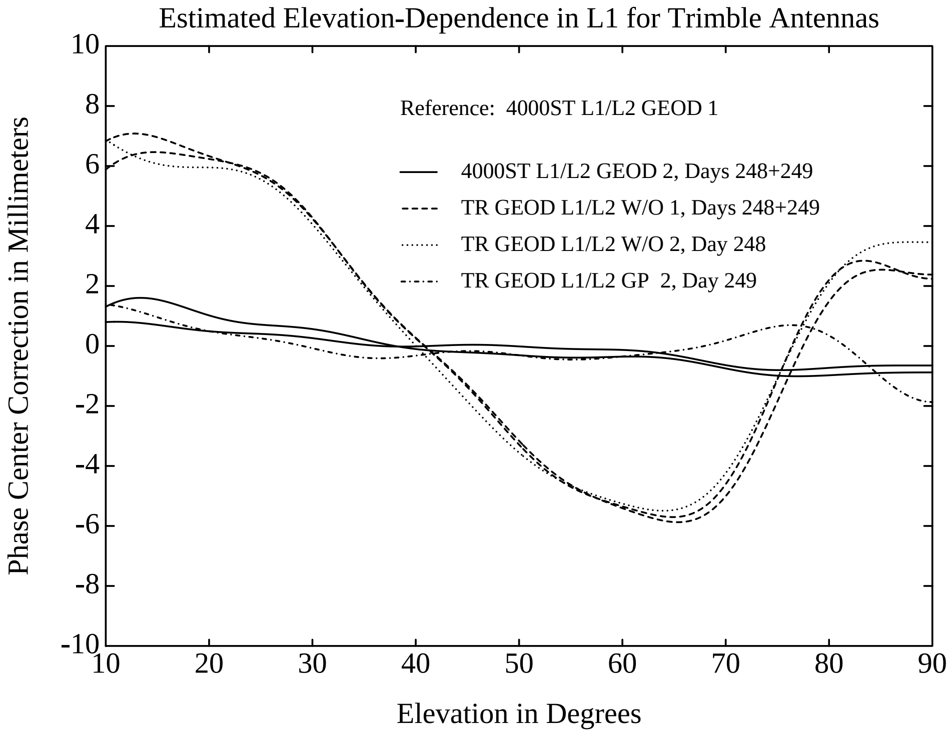
$$\delta\vec{r}_{A,ant} = \vec{r}_0 + \delta\phi(z,\alpha) \cdot \vec{e}_{z,\alpha}$$

$\vec{r}_0$	**Mean phase center offset** with respect to the antenna reference point (ARP, physical point).
$\delta\phi(z,\alpha)$	**Elevation** (and azimuth) **dependent phase center correction** with respect to the mean phase center $\vec{r}_0$
$\vec{e}_{z,\alpha}$	Unit vector in satellite direction

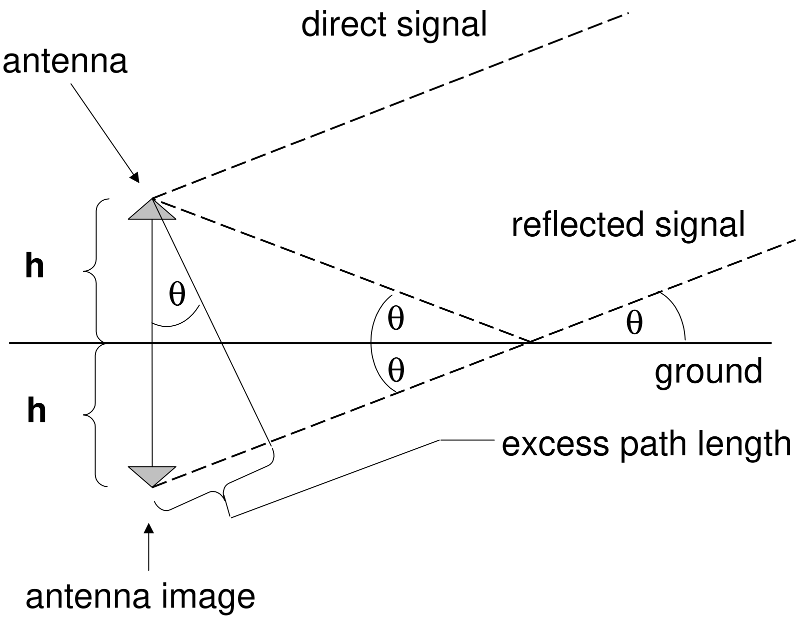


ANTENNA PHASE CENTER VARIATIONS (2)

- Two types of biases:
  - Combination of **different** antenna types: main effect in height, up to **10 cm**.
  - On long baselines even for **same** antenna type: main effect is a scale factor, up to **0.016 ppm**.
- **Phase center corrections** (offset and pattern relative to the Dorne Margolin T antenna) are available for most geodetic antenna types at the IGS information system.



MULTIPATH



- Multipath is the result of GPS signals, that are **reflected** from a surface near the antenna to the antenna. The GPS antenna then receives the direct as well as the indirect signal.
- This leads to systematic biases of **up to 20 m** in code and up to **a few centimeters** in phase measurements over time periods of minutes.
- Multipath effects are much more pronounced for **low elevation** satellites.
- Multipath is **difficult to model** because it depends heavily on the antenna environment.

6.

GPS PARAMETER SPACE

GPS PARAMETER SPACE

- **Site coordinates and velocities**: Geodynamics, geophyscis, positioning, navigation, surveying, reference frame, ...
- **Center of mass**: Monitoring Earth's center of mass, geophysics.
- **Satellite orbits**: Prerequisite for precise positioning, determination of Low Earth Orbiters (LEOs) for many future missions.
- **Earth orientation parameters**: Studies of Earth rotation (high-frequency, nutation), reference frame.
- **Satellite and receiver clocks**: High-precision time and frequency transfer (subnanosec level).
- **Tropospheric delays**: GPS receiver as accurate sensor for water vapor content in the atmosphere, atmospheric profiles with occultation technique.
- **Ionospheric delays**: Monitoring the ionospheric activity, generation of ionosphere maps (regional and global), ionosphere physics.