Improving the Orbit Estimates of the GPS Satellites *

T.A. Springer, G. Beutler, M. Rothacher
Astronomical Institute, University of Bern
Sidlerstrasse 5, CH-3012 Bern, Switzerland

Abstract

The Extended CODE Orbit Model, an empirical orbit model proposed by *Beutler et al.* [1994], was tested extensively since January 1996. Apart from six osculating Keplerian elements this orbit model consists of nine (instead of the conventional two) parameters to take into account the deterministic part of the force field acting on the satellites. Based on the test results an improved orbit parameterization is proposed in this article. The new orbit parameterization consists of the conventional two parameters plus three additional parameters, a constant and two periodic terms (a cosine- and a sine-term), in the X-direction to model the effects of the solar radiation pressure. Results based on one full year of routine orbit estimation, using the original and the new orbit parameterization, are presented to demonstrate the superiority of the new approach. An improvement of our orbit estimates with at least a factor of two is observed!

Keywords. GPS -- Solar Radiation Pressure - Orbit Estimation

1 Introduction

The Center for Orbit Determination in Europe (CODE), one of seven IGS Analysis Centers (AC's), is a joint venture of the following institutions:

- the Federal Office of Topography (L+T), Wabern, Switzerland
- the Institut Geographique National (IGN), Paris, France
- the Institute for Applied Geodesy (IfAG), Frankfurt, Germany
- the Astronomical Institute of the University of Berne (AIUB), Bern, Switzerland

The CODE is located at the AIUB and uses the Bernese GPS Software [*Rothacher and Mervart*, 1996], currently version 4.1, for GPS data processing.

Since the start of the International GPS Service for Geodynamics (IGS), on June 21, 1992, CODE has produced satellite ephemerides for all active GPS satellites and daily values for the Earth rotation parameters. Starting with January 2, 1994, all individual AC orbit (and clock) solutions are evaluated and combined into official IGS orbit/clock solutions by the IGS Analysis Coordinator on a weekly basis using procedures explained in [*Beutler et al.*, 1995; *Kouba*, 1995]. The IGS combinations/evaluations, summarized in weekly IGS reports, clearly demonstrate the steady improvements in both, precision and reliability, of the orbits for all AC's. The position RMS errors of the CODE orbits crossed the 10 cm level by the end of 1994. By the end of 1995 the CODE RMS errors had decreased to a level of 6 cm, by mid 1997, it reached the level of 4 cm!

Thanks to the enormous improvement of the orbit quality in only a few years time it became clear that the classical orbit model, characterised by eight parameters (6 initial conditions and 2 radiation pressure parameters), is not adequate to guarantee an orbit quality below the 10 cm level. Different AC's solved

* **reference:** *Accepted for publication in the Journal of Geodesy*, November 1998

this problem in different ways using either deterministic or stochastic approaches. At CODE the stochastic approach was selected to deal with model deficiencies of the classical orbit model. Starting June 4, 1995, small velocity changes, which we call pseudo-stochastic pulses, in the radial and along-track directions are estimated for *all* satellites at noon and midnight. It became clear by mid 1995, however, that the Extended CODE Orbit Model (ECOM), as proposed by *Beutler et al.* [1994] and used in the IGS orbit comparisons for the long-arc analysis [*Beutler et al.*, 1995], should *also* be capable of producing orbits below the 10 cm RMS level. Therefore, in early 1996, this deterministic model was fully implemented into the Bernese GPS Software and first experiences were gained [*Springer et al.*, 1996].

The first part of this extensive report presents the results of two different test series. In both series we systematically test different subsets of the parameters of the Extended CODE Orbit Model. In the first, stochastic, test series the pseudo-stochastic pulses are *always* estimated. In the second, deterministic, test series the pseudo-stochastic pulses are *not* estimated. The stochastic pulses, which are used in the CODE parameter estimation procedures, are well-suited to account for orbit model deficiencies [*Wu et al.*, 1991]. The estimation of such pulses, however, weakens the solution significantly because a relatively large number of additional parameters is estimated (for the CODE 3-day arcs 10 additional parameters per satellite). Based on the results of the stochastic test series an optimized orbit parameterization is found. This optimized parameterization consists of a subset of the nine parameters of the Extended CODE Orbit Model. The improved orbit parameterization was implemented into the official CODE contributions to the IGS on September 29, 1996.

The second part of this report analyses the differences between the routinely computed results, of both the original and new (optimized) orbit parameterizations, using a time span of one full year. SLR observations, covering the same one year time span, are used to study the quality differences between the two orbit solutions as well. The SLR data is especially suited for this purpose because it provides a truly independent check of the quality of the orbit estimates.

A discussion of the results, focussing on further improvement of both, the orbit model and orbit parameterization, concludes our report.

2 Solar Radiation Pressure Models

A priori the effect of the solar radiation pressure (RPR) on GPS satellites is modeled using the so-called ROCK models [*Fliegel et al.*, 1985, 1992]. The ROCK models are based on the size, shape and material properties of the GPS satellites and were developed to accurately model the effect of solar radiation. The models are not perfect; it is recommended to solve for a direct radiation pressure (scale-)term and a so-called ''Y-bias'' for precise orbit estimation [*Fliegel et al.*, 1992]. The expression ''Y-bias'' is justified because this constant bias characterizes an acceleration along the direction of the Y-axis of the satellite's body coordinate system. This body-fixed coordinate system is oriented with its +Z direction towards the Earth (along the satellite antennas). The Y-axis points along one of the solar panel beams and the third axis completes a right handed system and is positive towards the half plane that contains the Sun [*Fliegel et al.*, 1992].

2.1 The Extended CODE Orbit Model

In *Beutler et al.* [1994] the Extended CODE Orbit Model (ECOM) is discussed in detail, therefore only the basic characteristics are summarized here. The considerations behind the ECOM are similar to those underlying the Colombo model [*Colombo*, 1989]. The principal difference resides in the fact that the ECOM considers the Sun as the major ''error source'' for the orbits, whereas the gravity field of the Earth plays this role in the Colombo model. The Colombo model uses the radial, along- and cross-track directions as the three orthogonal directions whereas the D-, Y- and X-directions are used by the ECOM (definition see below). *Beutler et al.* [1994] demonstrated that the performance of the ECOM is superior to that of the Colombo model, which underlines that the solar radiation pressure is indeed the major error source for GPS satellite orbits. In the ECOM the acceleration $\vec{a}_{RPR}$ due to the solar radiation pressure (RPR) is written as:

$$\vec{a}_{RPR} = \vec{a}_{ROCK} + D(u) \cdot \vec{e}_D + Y(u) \cdot \vec{e}_Y + X(u) \cdot \vec{e}_X \tag{1}$$

where $\vec{a}_{ROCK}$ is the acceleration due to the ROCK model, and

$$\begin{aligned} D(u) &= D_0 + D_{Cu} \cdot \cos u + D_{Su} \cdot \sin u \\ Y(u) &= Y_0 + Y_{Cu} \cdot \cos u + Y_{Su} \cdot \sin u \\ X(u) &= X_0 + X_{Cu} \cdot \cos u + X_{Su} \cdot \sin u \end{aligned} \tag{2}$$

where D_0, D_{Cu}, D_{Su}, Y_0, Y_{Cu}, Y_{Su}, X_0, X_{Cu}, and X_{Su} are the nine parameters of the ECOM, and

$\vec{e}_D$ is the unit vector satellite-Sun, positive towards to Sun,
$\vec{e}_Y$ is the unit vector along the spacecraft's solar-panel axis,
$\vec{e}_X$ $\vec{e}_D \times \vec{e}_Y$, and
u is the argument of latitude of the satellite.

The ECOM is a generalization of the standard orbit model which uses only two parameters to account for the solar radiation pressure, namely D_0 and Y_0. Note that the Y-direction of the ECOM corresponds to the Y-direction of the body-fixed coordinate system.

3 Orbit Estimation Tests

The results of the two different test series are discussed. In the first, stochastic, test series we *always* estimate pseudo-stochastic pulses in the radial and along-track directions, whereas in the second, deterministic, test series we do *not* estimate pseudo-stochastic pulses.

The constant accelerations in the D- and Y-directions (parameters D_0 and Y_0) are always estimated, and the ROCK-T models, T10 for Block I and T20 for Block II and Block IIa satellites, are used as a priori model to account for the effects of solar radiation [*Fliegel et al.*, 1992].

Initial tests [*Springer et al.*, 1996] indicated that it is best not to solve for ''X-terms'', but to estimate the constant and periodic terms in the D- and Y-directions plus pseudo-stochastic pulses in the radial and along-track directions. A careful analysis of the proposed parameterization showed a significant degradation of the quality of the length-of-day (LOD) estimates.

The stochastic test series, consisting of more than 5 weeks of data (June 19 to July 27, 1996), was used to systematically analyse several parameter combinations of the ECOM with the aim to find the best orbit parameterization without compromising other (non orbit) parameters. For this stochastic test series we decided that periodic terms in a specific direction would only be estimated if the constant term in that direction was estimated, too. This means, e.g., that periodic terms in X were only estimated if the X_0 term was estimated, as well.

The deterministic test series, a 21-day data set (12--30 April, 1997), was used for a similar series of tests but now *without* the estimation of pseudo-stochastic pulses. Based on the results of the stochastic test series it was suspected that there are strong correlations between some of the parameters of the ECOM and the stochastic pulses. Therefore this deterministic test series should show the effects of individual parameters of the ECOM in a much clearer way. With this purely deterministic test series all possible combinations of the parameters of the ECOM were studied.

The general processing strategy in both test series was identical with that of the normal CODE routine IGS analysis [*Rothacher et al.*, 1995*a*, 1996]. The *only* differences between our test solutions and the official CODE solutions are the number and type of estimated orbit parameters.

Important characteristics of the CODE analysis are the *ambiguity fixing* and the *use of 3-day arcs*. At CODE about 80 to 90% of the ambiguities of all baselines shorter than 2000 km are resolved to their integer values using the so-called Quasi Ionosphere-Free (QIF) strategy [*Mervart*, 1995]. Ambiguity fixing strengthens the solutions significantly. Since the start of the IGS the CODE orbits have been based on 3-day arcs. The CODE arcs are relatively long compared to the 24--30 hour arcs used by most other IGS Analysis Centers. Both, ambiguity fixing and arc length, accentuate orbit model deficiencies.

3.1 Quality Assessment

To determine whether or not a certain orbit parameterization is acceptable we have to study the quality of *all* estimated parameters. Therefore, not only the orbit quality but also the overall quality of the solution, the quality of the station coordinates, and the quality of the Earth orientation parameters have to be analysed. Here we first describe our methods to assess the quality of the different estimated parameters. Subsequently these methods are applied to estimates of the two test series. The results are summarized in Tables 1 and 2 for the stochastic and deterministic test series, respectively, and will be discussed in the next section.

To assess the overall quality of the solution the mean of the a posteriori RMS error (of one single difference observation) over all 3-day solutions was computed. The value itself is not very meaningful but the ratios of these values of different test series give an indication whether the estimated orbit model parameters are absorbing (orbit-)errors or not. In 1993 and 1995 the RMS difference between the 1-day and 3-day solutions led us to the conclusion that the orbit model was not good enough. The mean RMS values of our different test series are given in the column labeled ‘‘SD rms’’ (Tables 1 and 2). Note that in the Bernese Software the post fit RMS of the residuals is formally converted to a single difference RMS on the L_1 frequency.

The formal errors of the estimated Earth Orientation Parameters (EOPs) for the X- and Y-components of the pole and for the LOD are indicators of their quality. Again, the actual values of these formal errors are not very meaningful but, the ratios of these quantities in different test series are informative. The formal errors of the LODs are of particular interest because they give an indication of the degree of correlation of orbit parameters with LOD: high formal errors indicate a high degree of correlation with orbit parameters and thus may lead to ‘‘noisy’’ LOD-estimates. The formal errors of the EOPs may be found in the columns labeled ‘‘EOP σ’’, where the units for X and Y are 10 mircoarcseconds, and for LOD the units are microseconds.

A good repeatability of station coordinates within the test series is an good indicator for the quality of the coordinate estimates, and a strong indication for a good orbit parameterization. If the orbits are not well modeled over the three days this will lead to distortions in the realization of the reference frame, which in turn will have a negative effect on station coordinates. The coordinate repeatabilities are given in the column labeled ‘‘CRD rms’’.

The orbit quality itself is assessed in two ways. The first method is based on a so-called *orbit overlap* test, i.e., the difference between the satellite positions from two (independent) solutions at a certain epoch is computed. For 1-day arcs there is exactly one epoch where the satellite positions in subsequent arcs can be compared without extrapolating the arcs. The end point of one arc corresponds to the starting point of the next arc (24:00 and 00:00 hours). With overlapping 3-day arcs we have a 48 hours overlap interval. At CODE usually the end of the middle day of a 3-day arc is compared with the start of the middle day of the next 3-day arc, e.g. using one single epoch. This method will be used here as well. The mean ‘‘3-dimensional’’ overlap, averaged over all 3-day arcs for all satellites, is computed. Only overlaps with position differences below 400 mm are used, larger differences are considered outliers, which implies that the number of overlaps, used to compute the mean, may slightly differ for different series. With the overlap RMS reaching the 50 mm level for the good solutions this outlier level is quite tolerant. The ratios of the overlap quality for different test series are informative.

The second method compares the orbit to the combined IGS orbit as produced by the IGS Analysis Coordinator [*Kouba and Mireault*, 1996]. In this comparison the orbit positions, as given in the precise orbit files with a 15 minute spacing, are compared using a 7-parameter Helmert transformation. The RMS error, over all satellite epochs, of this comparison is computed for the three individual days of the 3-day arcs. The mean RMS error, over all 3-day solutions, gives a good indication of the quality of the 3-day arcs. The mean RMS for the first and last day of the 3-day arcs are particularly well-suited as indicators for the orbit quality. In the case of a ‘‘bad’’ orbit model the satellite positions will deteriorate rapidly towards the end points of the arcs. The mean RMS differences for the three individual days of the 3-day arcs are given in the columns ‘‘Comparison with IGS Orbit’’. Also given is the mean scale difference for the middle day of the 3-day arcs.

4 Results

The results of the quality assessment, described in the previous section, are summarized in Tables 1 and 2 for the stochastic and deterministic test series, respectively.

The first column in Tables 1 and 2 is a solution identifier characterizing the orbit parameterization. The

Table 1: Results of orbit tests using the Extended CODE Orbit Model *including* the estimation of pseudo-stochastic pulses. Based on data from June 19 to July 27, 1996.

Sol.	RPR-Model						SD	EOP σ			CRD	Overlap		Comparison with			
ID	Const.			Periodic			rms	X	Y	L	rms	Nr	rms	IGS Orbit (mm, ppb)			
	D	Y	X	D	Y	X	mm	10μas		μs	mm		mm	D1	D2	D3	Sc
R3	X	X	-	-	-	-	3.84	3	3	1	7.6	763	145	127	69	124	.0
Dp	X	X	-	X	-	-	3.35	2	3	4	6.1	770	51	73	65	70	.0
Yp	X	X	-	-	X	-	3.44	2	3	1	6.5	-	-	90	70	90	.0
DY	X	X	-	X	X	-	3.30	2	3	4	6.1	769	48	72	65	70	.0
X0	X	X	X	-	-	-	3.69	3	3	1	6.7	767	130	125	69	121	.3
XDp	X	X	X	X	-	-	3.33	2	3	6	6.1	-	-	90	80	90	.7
XYp	X	X	X	-	X	-	3.35	2	3	2	6.4	770	51	78	69	77	.3
X3	X	X	X	-	-	X	3.31	2	3	2	6.0	770	49	71	66	70	.3
XDY	X	X	X	X	X	-	3.28	2	3	7	6.1	765	88	91	82	93	.3
XDX	X	X	X	X	-	X	3.29	2	3	7	6.1	-	-	90	80	90	.4
XYX	X	X	X	-	X	X	3.28	2	3	2	6.1	769	53	75	69	75	.4
ALO	X	X	X	-	-	X	3.33	2	3	2	6.0	768	53	74	71	75	.3
RAD	X	X	X	-	-	X	3.61	2	3	1	6.6	724	97	106	92	114	.4

next six columns show which parameters of the ECOM were estimated (X) or constrained to zero (-). Note that the periodic terms effectively consist of two parameters (a cosine and a sine term). The other columns were described in detail in the quality assessment section.

4.1 Stochastic Orbit Tests

For the stochastic test series (Table 1) all mean values are based on the complete set of 39 3-day solutions created for this test series. Note that the solutions labeled ''ALO'' and ''RAD'' will be discussed in the next section.

Some tests were not performed over the complete length of 39 days, because the orbit parameterization was not deemed adequate at an early stage in the test run. Solution ''Yp'' was stopped because the quality of the estimated orbits was not satisfying. The tests with solutions ''XDp'' and ''XDX'' were stopped because the LOD estimates were very noisy. For these incomplete solution series no orbit overlap test was performed and the mean of the orbit comparison was rounded to an integer number of centimeters.

Note that solution ''R3'' was the official CODE contribution to the IGS from June 4, 1995 to September 29, 1996 (GPS weeks 804 to 873). The orbit comparison of the R3-orbit for the middle day (D2) of the arc with the IGS orbits should be considered with care: the R3-solution has contributed to the combined IGS orbit with a heavy weight, because it usually was considered as one of the best solutions by the IGS combination scheme. Therefore the orbit comparison of the R3-orbit with the IGS orbit is, to a certain extent, a comparison with itself.

Clearly, *all* test solutions perform better in *all* aspects, except for the LOD estimates, than the former official CODE solution (R3). This is not surprising because we found out some time ago that the R3-parameterization is not adequate to model the satellite orbits over three days. All other solutions have at least one additional orbit parameter which is capable of absorbing some of the orbital errors.

From Table 1 we conclude that there is a periodic signal and a small constant bias in the X-direction. There is also a periodic signal in the Y-direction but it is weaker than the periodic signal in the the X-direction. Furthermore, there might be a periodic signal in the D-direction but this parameter correlates very strongly with other parameters and therefore it cannot be determined very well.

Table 1 reveals that solutions Dp, DY, XYp, X3, and XYX give the best results. In fact, solution DY was known to give good results for a long time [*Springer et al.*, 1996]. However, both the Dp- and DY-solutions, show a significant increase of the formal error of the LOD estimates. It can be shown, in

Table 2: Results of orbit tests using the Extended CODE Orbit Model *without* estimation of pseudo-stochastic pulses. Based on data from April 12 to April 30, 1997.

	RPR-Model						SD	EOP σ			CRD	Overlap		Comparison with			
	Const.			Periodic			rms	X	Y	L	rms	Nr	rms	IGS Orbit (mm, ppb)			
ID	D	Y	X	D	Y	X	mm	10μas		μs	mm		mm	D1	D2	D3	Sc
R3	X	X	-	-	-	-	3.44	2	2	1	6.4	367	109	122	68	119	.0
X3	X	X	X	-	-	X	2.94	2	2	2	5.9	367	43	54	50	56	.2
RS	X	X	-	-	-	-	5.58	3	3	1	11.6	189	253	437	131	440	.0
DS	X	X	-	X	-	-	5.27	3	3	4	11.7	190	261	409	129	408	.1
YS	X	X	-	-	X	-	3.80	2	2	1	8.2	366	101	179	77	181	-.1
XS	X	X	-	-	-	X	3.23	2	2	0	5.9	366	75	84	59	79	.0
AS	X	X	-	X	X	-	3.14	2	2	3	6.5	366	70	94	61	84	.0
BS	X	X	-	X	-	X	2.98	2	2	3	5.6	366	53	60	52	56	.0
CS	X	X	-	-	X	X	3.08	2	2	1	5.8	366	71	75	61	72	.0
ZS	X	X	-	X	X	X	2.97	2	2	5	5.6	366	57	62	56	62	.0
RT	X	X	X	-	-	-	5.50	3	3	1	11.7	194	258	431	131	434	.1
DT	X	X	X	X	-	-	5.14	3	3	5	12.6	165	278	403	161	412	.5
YT	X	X	X	-	X	-	3.72	2	2	1	8.0	366	101	176	78	178	.0
XT	X	X	X	-	-	X	3.15	2	2	1	5.7	366	68	79	57	76	.2
AT	X	X	X	X	X	-	3.08	2	2	4	6.6	362	95	97	76	92	-.6
BT	X	X	X	X	-	X	2.97	2	2	4	5.5	361	74	67	60	67	-.3
CT	X	X	X	-	X	X	3.01	2	2	1	5.7	366	63	69	59	67	.2
ZT	X	X	X	X	X	X	2.96	2	2	4	5.6	361	78	71	65	72	.2

a similar way as it was done in *Rothacher et al.* [1995*b*], that this is caused by the periodic terms in the D-direction. This aspect, however, is beyond the scope of our present investigation. The performance of the XYp-solution is slightly worse in all aspects when compared to the X3-solution. The quality of the XYX-solution is very similar to that of the X3-solution. Because the XYX-solution solves for two additional parameters (the periodic terms in the Y-direction) without significant improvement, solution X3 was chosen as ''the optimal solution''.

The X3-solution is significantly better than the R3-solution. The orbit overlap discrepancy drops from 145 to 49 mm, a dramatic improvement by almost a factor of three! An improvement by a factor of two is observed in the orbit comparisons for the first (D1) and last day (D3) of the 3-day orbit arcs. Even the middle day (D2) compares better to the combined IGS orbit which is striking evidence for the quality of the X3-solution (keeping in mind that the R3-solution contributed to the IGS combined orbit)! Furthermore, the improvement of the repeatability of the station coordinates is surprising (only small differences were expected here). The only side effect encountered so far is the small change in the scale of the orbit. The X3-orbits are slightly ''higher'' compared to the R3- and IGS orbits. This scale difference is observed for *all* solutions in which the X_0 term is estimated.

Based on these results the X3-solution was defined as the official CODE contribution to the IGS since September 29, 1996 (GPS week 873).

4.2 Deterministic Orbit Tests

The results of the deterministic test series in Table 2 are based on all 21 3-day solutions. For reference the normal routine solutions, R3 and X3, are also included. For the routine solutions pseudo-stochastic pulses are estimated, of course. Note that for this time frame the X3-solution contributed heavily to the IGS orbit which is used for the orbit comparisons. Therefore, orbit comparisons of the X3-orbit with the IGS orbit are not comparisons of two independent orbits.

The results in Table 2, especially those related to solution BS, show that it is possible to get good orbits using a purely deterministic orbit model. The differences between the deterministic BS-solution and the stochastic X3-solution are relatively small and are, most likely, due to a few (eclipsing) satellites with modeling problems. The major disadvantage of the BS-solution is the increased formal error of the LOD. In fact the LOD estimates are significantly less precise than those of the X3-solution. This makes this solution unacceptable at this stage for our routine IGS operations.

From Table 2 we conclude that there are significant periodic signals in the X- and Y-directions, with the strongest signals in the X-direction. The existence of a constant bias in the X-direction (compare solutions RS and RT) and periodic signals in the D-direction (compare solutions RS and DS) are not evident from these results. Based on the differences between solutions R3 and X0 in Table 1 a constant bias in the X-direction was expected. The results of the purely deterministic orbit estimates given in Table 2 do not support this expectation. Nevertheless, if periodic X-terms are estimated the constant X-term does improve the solutions (compare solutions XS and XT). We conclude that the selection of the X3-solution, based on the results of Table 1, was correct although the estimation of the X_0-term is debatable.

5 Results based on 1-Year of Routine Orbit Estimation

Although the CODE contributions to the IGS are based on the new solution starting September 29, 1996 (GPS week 873) this solution type was already routinely computed since August 4, 1996 (GPS week 865). Apart from this new official X3-solution, the original R3-solution was continued in order to study the differences between the two solution types. This allows us to study the differences in the estimated deterministic radiation pressure parameters, and the estimated pseudo-stochastic pulses of the two solution types over one full year of routine orbit estimation.

Furthermore, we study the orbit differences by performing the previously discussed orbit overlap and orbit comparison tests. The resulting EOP series of the two orbit series will be compared and, finally, we will use SLR observations to the two GPS satellites equipped with Laser reflector arrays, to study the quality of the two orbit parameterizations.

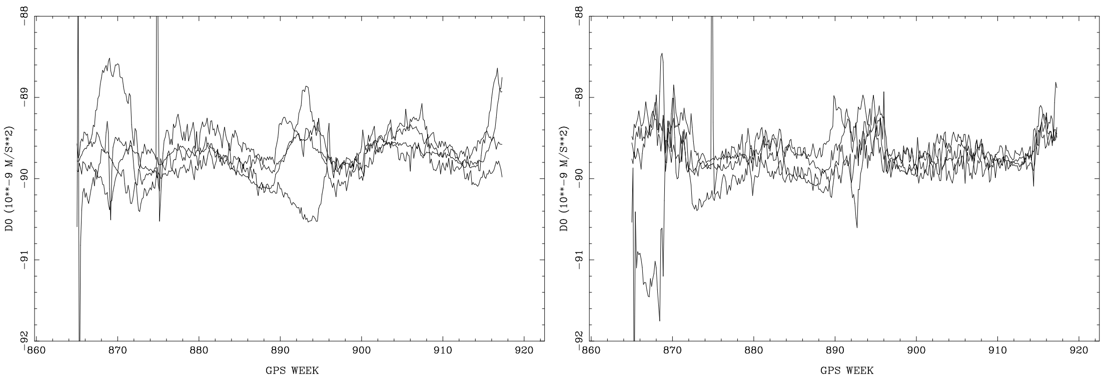
Notice that for the orbit estimation (using 3-day arcs) two stochastic pulses (radial and along-track) per satellite are estimated at 5 different epochs (noon and midnight each day except arc boundaries). Contrary to the initial conditions and the radiation pressure parameters, which are estimated completely ''free'', the pseudo-stochastic pulses are constrained to $1 \cdot 10^{-6} m/s$ and $1 \cdot 10^{-5} m/s$ for the radial and along-track components respectively. The RPR-parameters are estimated as constants over the entire period of the 3-day arcs. The epoch of the RPR-estimates therefore corresponds to the middle of the arc, i.e., to noon UT of the second day of the 3-day arc.

It is well known that the orbits of GPS satellites are very difficult to model during eclipse phases and therefore we may expect some anomalous behaviour in the orbit estimates during the eclipse seasons. Satellites are eclipsed when they pass through the Earth shadow during the orbital revolution. The satellite is eclipsed for a maximum of about 55 minutes when the Sun is lying directly in the orbital plane. Each satellite has ''eclipse seasons'' twice per year. The major problem with eclipsing satellites is the fact that the attitude of the satellite is incorrect when it returns into the sunlight. It takes the satellite about 30 minutes after eclipse exit to return to its nominal attitude [*Fliegel et al.*, 1992; *Bar-Sever*, 1995].

5.1 Time Series of Deterministic and Stochastic Parameters

Some effects in the RPR-estimates are related to the orbital plane which is why we combine the RPR-estimates pertaining to one orbital plane in our discussion. Furthermore, the absolute values of the direct solar radiation pressure (term D_0) are satellite block-type specific. The mean values for the constant D_0 of the Block I, II and IIa satellites differ significantly. In the main text we will therefore confine ourselves to orbital plane C containing satellites PRN 3, 6, 7, and 31. Orbit plane C is the only plane containing only one type of satellites, namely Block IIa. PRN 28, also a Block IIa satellite in plane C, was deactivated for a long period within the test interval; therefore we do not include this satellite here.

The estimates of the direct solar radiation pressure acceleration, D_0, may be inspected in Figure 1 for the R3- and X3-orbit solutions. The eclipse periods for orbital plane C, GPS weeks 865--872 and 889--896, are easily recognized in Figure 1. As expected the D_0-estimates, of both the R3- and X3-solutions, are



(a) Orbit Model R3

(b) Orbit Model X3

Figure 1: Estimated direct solar radiation pressure acceleration using the two different CODE orbit parameterizations. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.

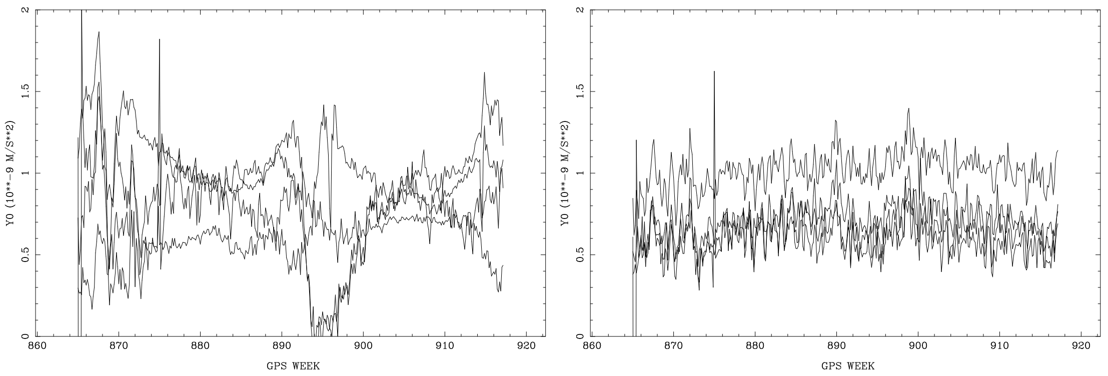
significantly different and noisier during both eclipse periods compared to the non-eclipsing intervals. The D_0-estimates of the X3-solution seem to ‘‘behave’’ better than the R3-estimates, particularly outside the eclipse seasons. The R3-estimates show a small signal with a period of about half a year which cannot be detected in the X3-estimates. During the eclipse seasons the X3-estimates are noisier than the R3-estimates but the R3-estimates show some relatively large systematic effects.

The daily estimates of the so-called Y-bias, Y_0, from both the R3- and X3-orbit solutions are shown in Figure 2. We see significant differences between solutions R3 and X3. The estimates of the R3-solution seem to be erratic during eclipse periods and they show a periodic behaviour between eclipses. In the X3-estimates it is difficult to detect the eclipse periods. The X3-estimates are more or less constant over the entire year, whereas the R3-estimates show large satellite specific signals. We conclude that the Y-bias estimates of the X3-solution are superior to those of the R3-solution, in particular because it seems plausible that the Y-bias is constant for all satellites. This indicates that the orbit model deficiencies are reduced in the X3-solution.

The estimated stochastic pulses in the along-track direction are given in Figures 3. Figure 3 reveals that the along-track estimates are much more consistent for the X3- than for the R3-solution. Figure 4 shows the estimated radial pulses from both the R3- and X3-solution series, the difference between the solutions is striking, indeed! Notice the much larger scale of Figure 4 compared to Figure 3. Clear signals are present in the radial pulses of the R3-solutions, whereas the X3-estimates are (apart from noise) almost ideally zero. Clearly there is much more signal (more modeling problems) in the estimated pseudo-stochastic pulses of the R3-solution in both, the along-track and radial directions. The pulses in the radial direction of the X3-solution seem to be noise only. Considering the fact that pseudo-stochastic pulses are meant to absorb orbit model deficiencies it is clear that the modeling deficits are significantly reduced in the X3-solution.

Figure 4 seems to indicate that it is no longer necessary to estimate the radial pseudo-stochastic pulses. To test whether or not it is still necessary to solve for the radial pulses two additional solutions were made using the same data which was used for the stochastic test series. These two additional solutions are identical to the X3-solution, except that in one case only along-track pulses were estimated and in the other case only radial pulses. These solutions are labeled ‘‘ALO’’ and ‘‘RAD’’, respectively, and the results are included in Table 1.

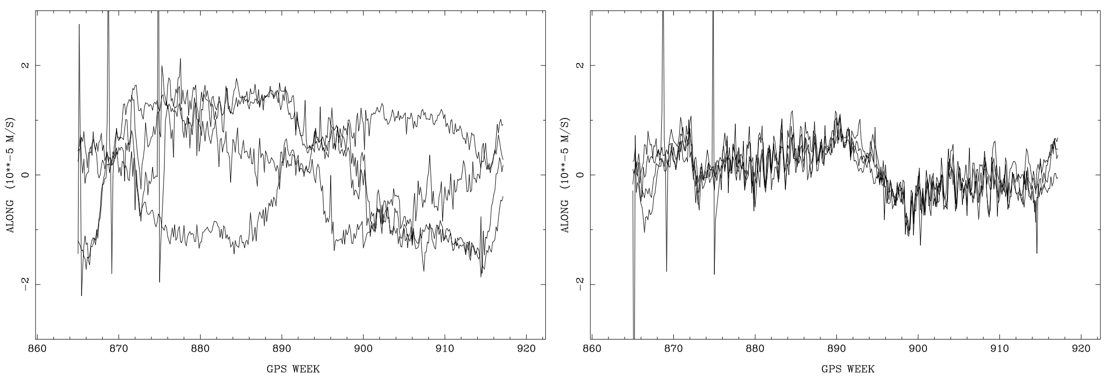
Not estimating radial pulses, solution ‘‘ALO’’, has almost no influence on the quality of the (orbit) estimates, as was expected from Figure 4. The slight degradation is due to some problematic (eclipsing) satellites for which the estimation of the radial pulses is useful. Not estimating along-track pulses, solution ‘‘RAD’’, leads to a significant degradation of the estimates, as was expected based on the remaining signals observed in Figure 3. Note, however, that the results are still better than the results obtained with



(a) Orbit Model R3

(b) Orbit Model X3

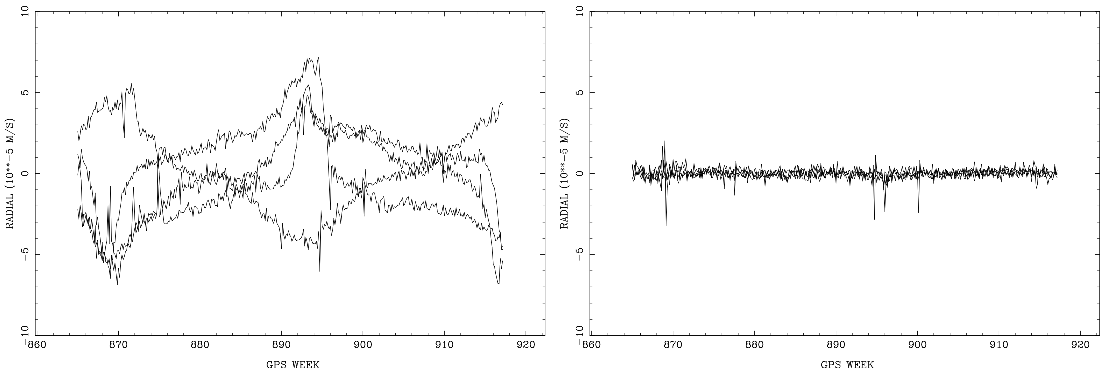
Figure 2: Estimated ''Y-bias'' using the two different CODE orbit parameterizations. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.



(a) Orbit Model R3

(b) Orbit Model X3

Figure 3: Estimated Along-track pulses (at noon) using the two different CODE orbit parameterizations. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.



(a) Orbit Model R3

(b) Orbit Model X3

Figure 4: Estimated Radial pulses (at noon) using the two different CODE orbit parameterizations. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.

Table 3: Results of orbit overlap tests for the R3- and X3-solutions using one full year of orbit estimates.

	Mean Orbit Overlap (mm)									
	Non-Eclipsing Sat.					Eclipsing Sat.				
Sol	# overlaps	rad	alo	out	tot	# overlaps	rad	alo	out	tot
R3	6954	39	69	84	116	1511	97	122	111	191
X3	6988	21	34	31	51	1804	25	39	28	54

the original R3-orbit!

Figure 5 shows the estimates of the constant force in the X-direction (X_0) from the X3-solution. Although the estimates are quite noisy a small mean offset of about $-0.4 \cdot 10^{-9} m/s^2$ is observed for all satellites. Only during the eclipse phases the X_0-estimates are quite erratic.

Figure 6 shows the estimates of the cosine and sine terms in the X-direction. A strong annual signal is observed for all satellites with an amplitude of about $2 \cdot 10^{-9} m/s^2$. The cosine estimates are very noisy during the eclipse phases of the satellites, whereas the sine terms seem to be very well-established during the eclipse. The annual signal is probably due to the annual variation in the orientation of the orbital plane with respect to the Sun. This is supported by a similar behaviour of all orbital planes (with 60^o phase differences between subsequent orbital planes) which indicates that these accelerations are caused by the solar radiation pressure. In analogy to the name Y-bias, the constant and periodic forces observed in the X-direction are tentatively called ‘‘X-bias’’.

5.2 Orbit and EOP Differences

The same orbit overlap test as performed for our two test series was applied to the two routine orbit series R3 and X3. The results are shown in Table 3 where we distinguish between eclipsing and non-eclipsing satellites. The overlap discrepancies in the radial, along-, and cross-track (rad, alo and out) directions are given in addition to the total discrepancies.

The 3-dimensional overlap discrepancy shows a similar behavior as in Tables 1 and 2; an improvement by a factor 2 to 3 is seen for the X3-solution relative to the R3-solution. It is interesting that the differences are even more pronounced for the eclipsing satellites. The new parameterization seems to absorb a significant portion of the unmodeled forces during eclipse periods. In fact, for the X3-solution the overlap statistics are only slightly worse for eclipsing satellites than for the non-eclipsing satellites.

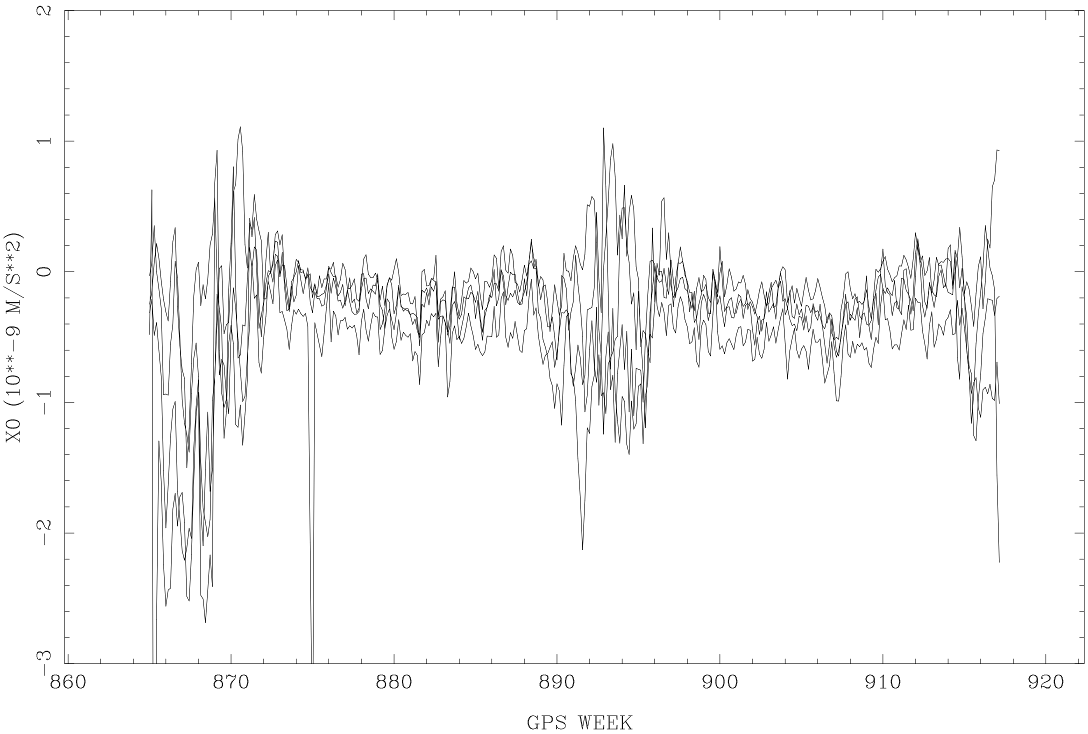
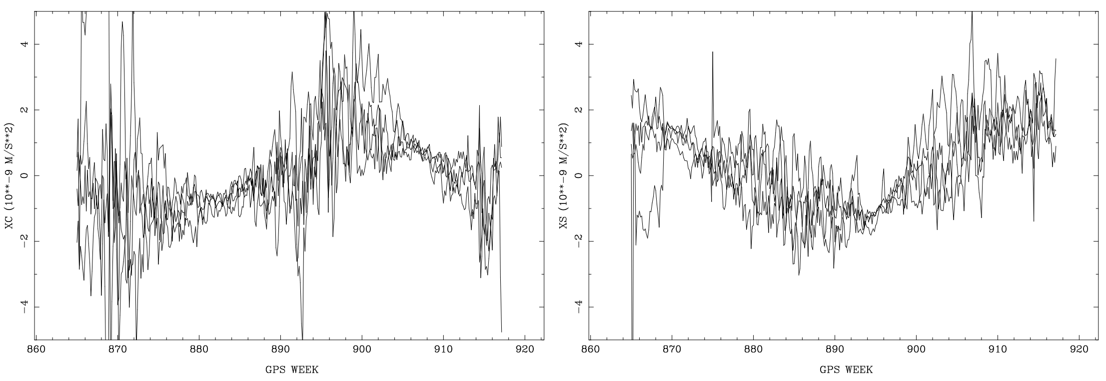


Figure 5: Estimated constant acceleration in the X-direction (X_0) using the new CODE orbit parameterization. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.



(a) Cosine

(b) Sine

Figure 6: Estimated periodic cosine and sine terms in the X-direction using the new CODE orbit parameterization. Only PRNs 3, 6, 7, and 31 in orbital plane C are shown.

Table 4: Results of EOP comparison of the R3- and X3-EOP estimates with the IERS Bulletin A pole series.

	X-pole (mas)		Y-pole (mas)		LOD (μs/day)	
Sol	MEAN	RMS	MEAN	RMS	MEAN	RMS
R3	0.31	0.14	-0.02	0.18	-15.5	21.7
X3	0.32	0.13	-0.04	0.17	15.8	26.6

In order to study whether there are significant orientation differences between the R3- and X3-orbits a seven parameter Helmert transformation (orbit comparison) was performed between the two solutions. The only significant difference was found in the scale of the orbits. A mean scale difference of 0.2 ppb is observed (the X3-orbits having a larger scale than the R3-orbits). No significant differences were encountered in the translation and rotation parameters. The scale difference was already observed in the results of the stochastic test series in Table 1, and it was shown to be due to the estimation of the constant acceleration in the X-direction. The mean RMS errors per satellite coordinate of the orbit comparisons is about 50 mm (reflecting the relatively large differences between the two orbit solutions).

Finally we compared the estimated Earth orientation parameters (EOP) of the R3- and X3-series to the official IERS Bulletin A EOP series. The results, the mean offset and the RMS of the X-, and Y-pole components and the LOD, can be inspected in Table 4. The only significant difference is found in the LOD estimates. Here a completely different offset w.r.t. the Bulletin A series is observed, in fact, the mean value has changed sign! We assume that the change of the orbit scale and the change of the LOD bias are correlated. Tentatively this may be explained as follows. Due to the increased scale of the X3-orbits (observed in the orbit comparisons) the orbit revolution period increases according to Kepler's law. To preserve the geometry between stations and satellites the rotation rate of the Earth has to slow down accordingly, which leads to an increased LOD.

5.3 Orbit Differences Based on SLR Observations

On August 30, 1993, PRN5 was launched. This particular Block IIa satellite carries a small panel of optical retroreflectors. Consequently this satellite may be tracked by the centimeter-accuracy satellite laser ranging (SLR) stations. On March 10, 1994, GPS satellite PRN6 was launched carrying an identical retroreflector package [*Degnan and Pavlis*, 1994; *Pavlis and Beard*, 1995]. The SLR observations of the two GPS satellites equipped with the Laser reflector array provide an unique possibility to evaluate the accuracy of GPS ephemerides. The SLR data are completely independent from the GPS data!

We use the SLR observations of the GPS satellites to evaluate the orbit estimates of our two different solutions (R3 and X3). The difference between the observed range (SLR observations) and the computed ranges is determined. The ranges are computed by taking the orbit positions from our orbit estimates. The SLR station positions are taken from the ITRF'94, to be consistent with the reference frame of the orbits, and the tropospheric delay is modeled using the Marini Murray model. Similar procedures were adopted by [*Zhu et al.*, 1997; *Watkins et al.*, 1996]. Note that there is no parameter estimation involved in this procedure.

Due to the high altitude of the GPS satellites, the angle between the vector from the SLR-observatory to the GPS-satellite and the vector from the center of mass of the Earth to the GPS-satellite is at maximum 14^o. This means that the SLR observations are nearly in the radial direction, and therefore the SLR observations mainly provide information about the radial orbit errors [*Watkins et al.*, 1996].

Results of the external check of the R3- and X3-ephemerides using the SLR observations are given in Table 5. All SLR data available in the year for which we have both, the R3- and X3-solutions, is used. SLR residuals exceeding the 300 mm were considered as outliers and removed. The residuals are given separately for the eclipse and non-eclipse periods of the two satellites. The number of accepted SLR observations is given in the first column, the mean offset of the residuals and the RMS errors (around the mean value) are given in the next two columns. We see that the number of accepted SLR observations is slightly different for the two solutions: 17 observations more for the X3-solutions for the eclipse periods

Table 5: Results of SLR Orbit Check.

	SLR O-C Residuals Statistics (mm)					
	Non-Eclipse period			Eclipse period		
Sol	#Obs	Mean	RMS	#Obs	Mean	RMS
R3	4741	-64	51	2797	-64	74
X3	4741	-72	51	2814	-72	74

of the two satellites which, however, is insignificant. Secondly, we see that the RMS errors of the SLR residuals is identical for the two solutions whereas the mean offset differs by 8 mm. The negative sign of the mean offsets of both solutions means that the SLR observations are short compared to the expected range.

The scale difference between the R3- and X3-orbit solutions (0.2-0.3ppb), observed in the orbit comparison, gives a good explanation of the difference in the mean offset of the SLR residuals. With the semi-major axis of the near circular GPS orbits of 26500 km, the observed orbit scale difference leads to a difference in radius of about 5-8 mm. This corresponds quite well with the observed mean offset difference between the SLR residuals of the R3- and X3-solutions. However, this does not explain the absolute value of the SLR residual, of 60-70 mm. This offset, also reported by, e.g. [*Watkins et al.*, 1996], is unexplained at present. There are a few possible explanations for this bias like e.g. the position of the SLR reflector on the GPS satellite, erroneous SLR station coordinates, an incorrect value of the Earth's gravitational constant (GM). However, a detailed investigation of this bias is beyond the scope of this paper.

The similarity of the RMS errors of the SLR residuals for the R3- and X3-orbits seems a bit disappointing because we expected a significant difference based on the previously shown superiority of the X3-solutions, especially for the eclipsing satellites. If, however, we look at the overlap differences in the radial direction in Table 3 we see that for the non-eclipsing satellites the radial overlaps are 39 and 21 mm for the R3- and X3-solutions respectively. These are relatively small values compared to the mean offset and RMS of the SLR residuals. Therefore we conclude that the RMS of the SLR residuals, for the non-eclipse periods of the satellites, is dominated by other error-sources than the (radial) orbit errors. In Table 3 we see that for the eclipsing satellites the radial overlaps are 97 and 25 mm for the R3- and X3-solutions, respectively. The RMS of the SLR residuals for the eclipse periods of both satellites, (74 mm for R3 and X3, Table 5), does not reflect the large difference between the R3- and X3-solutions. Although the RMS of the SLR residuals does reflect the degraded orbit precision during the eclipse periods it does not reflect the much more degraded accuracy of the R3-solution compared to the X3-solution. This is somewhat surprising!

6 Summary and Outlook

Based on our stochastic test series with the Extended CODE Orbit model we have seen that the estimation of the constant and periodic terms in the X-direction, in addition to the estimation of the constant terms in the D- and Y-direction, and the pseudo-stochastic pulses in the radial and along-track directions, significantly improves the orbit quality. The orbit estimates using this new parameterization are a factor of two to three better than with the original parameterization! As a direct consequence the new orbit parameterization is used for the generation of the CODE contributions to the IGS since September 29, 1996.

Our deterministic test series confirmed that, from the three directions in which periodic terms can be estimated with the Extended CODE Orbit Model, the periodic terms in the X-direction most significantly reduce the orbit model deficiencies. Evidence was presented that the periodic signals in the Y-direction reduce the orbit model deficiencies as well. The periodic signals in the X-direction, however, are more important than those in the Y-direction. The estimation of the periodic Y-terms, in addition to the constant D-, Y-, and periodic X-terms, did not clearly improve the orbit estimates. It is encouraging, however, that these terms can be estimated simultaneously without degradation of any of the other estimated parameters.

The deterministic test series also showed that an purely deterministic orbit parameterization, consisting out of the constant terms in the D- and Y-directions plus periodic terms in the D- and X-directions, can give excellent orbit estimates. Due to the degradation of the LOD estimates this deterministic orbit model is currently not used.

The results based on one year of routine orbit estimates using the original and new orbit parameterizations showed that the behaviour of the estimated stochastic and deterministic orbit parameters was much improved with the new orbit parameterization. This is true in particular for the Y-bias and the radial pseudo-stochastic pulses. Especially the improvement of the estimated stochastic orbit parameters shows that the orbit model was much improved with the new parameterization. Tests without estimating the radial pseudo-stochastic pulses showed that these pulses have no significant influence on the quality of the (orbit) estimates.

Based on the stochastic test series the existence of a constant acceleration in the X-direction was expected. The results of the deterministic test series, however, did not confirm this. Nevertheless a small improvement was seen when estimating the constant term in the X-direction in addition to the periodic terms in the same direction. The estimates of the constant X-term, based on one year of routine orbit estimation (Figure 5), revealed a small mean offset for the constant acceleration in the X-direction. Furthermore the results of the full year of routine orbit estimates showed that for both, the sine and cosine terms in the X-direction, a strong annual signal with an amplitude of approximately $2 \cdot 10^{-9} m/s^2$ exists for all satellites. In analogy to the name Y-bias, the constant and periodic forces observed in the X-direction are tentatively called ‘‘X-bias’’.

With the advent of the new orbit parameterization the Y-bias estimates have become much more stable. It is now possible to introduce good a priori values for the Y-bias without estimation or with tight constraints on the Y-bias parameter. A similar procedure can be attempted for the constant D- and X-terms, although the estimates during eclipse are erratic for both parameters. The strong annual signal observed in the estimates of the periodic terms in the X-direction, which is probably caused by the annual rotation of the orbital plane with respect to the Sun, makes it impossible to introduce a constant for these terms. Therefore our a priori model should be enhanced to account for this annual signal. This should lead to a constant value (bias) for both, the cosine and sine terms. In this way we expect to obtain a good, deterministic, radiation pressure model.

The remaining signal observed in the estimates of the along-track pseudo-stochastic pulses indicates that with the new improved orbit parameterization still some orbit model deficiencies remain. Tests without estimating the stochastic pulses confirmed this, indicating that, although the orbit estimates are much improved, further improvement is still possible.

The agreement between observed ranges, based on the SLR data, and computed ranges, based on the GPS satellite orbits and the SLR station positions, revealed a mean bias of 60 to 70 mm. The SLR ranges are short compared to the computed ranges. This bias, for which there is no good explanation at present, should be investigated in the near future. However the RMS agreement (around the mean) was found to be as low as 50 mm RMS, which is very encouraging.

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