A new Solar Radiation Pressure Model for the GPS Satellites *

T.A. Springer, G. Beutler, M. Rothacher
Astronomical Institute, University of Bern
Sidlerstrasse 5, CH-3012 Bern, Switzerland

Abstract

The largest error in currently used GPS orbit models is due to the effect of solar radiation pressure. Over the last few years many improvements were made in modeling the orbits of GPS satellites within the IGS. However, most improvements were achieved by increasing the number of estimated orbit and/or solar radiation pressure parameters. This increase in the number of estimated satellite parameters weakens the solutions of all estimated parameters (not only orbit parameters). Due to correlations the additional orbit parameters may introduce biases in other estimated quantities like, e.g., the length of day.

We present a recently developed solar radiation pressure model for the GPS satellites. This model is based on experiences and results gained at the Center for Orbit Determination in Europe (CODE) in the context of its IGS activities since June 1992. The performance of the new model is almost an order of magnitude better than that of the existing ROCK models. It also allows a reduction of the number of orbit parameters that have to be estimated!

Introduction

The largest non-gravitational effect on the GPS satellite orbits is solar radiation pressure (RPR). Table 1, which shows the effect of different perturbations on the GPS orbits, is based on the difference between two numerically integrated orbits over one day where in one orbit the perturbation effect considered is turned on, in the other turned of. Both integrations start from identical initial conditions. The RMS difference was computed using the full 24-hour arc length and all satellites (using the full satellite constellation of January 1, 1998). It can be seen that the size of the perturbation caused by the solar radiation pressure is only exceeded by the effects of the Earth oblateness, the gravitational attraction by Sun and Moon, and the lower harmonics of

*reference: Accepted for publication in *GPS Solutions*, ©1998 John Wiley & Sons, Inc *in press*

the Earth's gravity field. The results clearly show that an accurate solar radiation pressure model for the GPS satellites is as important as an accurate gravity model of the Earth.

Table 1: Effect of different perturbations on the GPS satellites over 24 hours

Perturbation	Magnitude (m)			
	Radial	Along	Cross	Total
Earth oblateness	1335	12902	6101	14334
Moon (gravitation)	191	1317	361	1379
Sun (gravitation)	83	649	145	670
C(2,2), S(2,2)	32	175	9	178
Solar Radiation Pressure	29	87	3	92
C(n,m), S(n,m) (n,m=3..8)	6	46	4	46

The basis of the RPR models currently used was developed by Rockwell International, the spacecraft contractor for the Block I, II and IIa satellites [*Fliegel* et al., 1992]. The computer programs that embody this model became known as ROCK4 for Block I [*Fliegel* et al., 1985], and ROCK42 for Block II and IIa satellites [*Fliegel and Gallini*, 1989]. They are also known as ''Porter models''. The ROCK models are expressed in the satellite-fixed coordinate system. This system has its origin in the center of mass of the satellite. Its Z-axis points in the direction of the center of mass of the Earth, and therefore along the satellite antennas. The X-axis is positive toward the half plane that contains the Sun and the Y-axis completes a right-handed system and points along one of the solar panel axis.

For high precision geodetic work it is necessary to estimate a scale term and an acceleration in the Y-direction, the Y-bias, in addition to using the ROCK model. The ROCK model, therefore, only serves as a priori information. Both, the scale term and the Y-bias, are parameters which are supposed to vary slowly in time. Although the cause for the Y-bias is unknown, its effect on the orbit is significant. The claimed accuracy of the ROCK models is about 3% of the full effect. Taking the nominal value of $1 \cdot 10^{-7} m/s^2$ for the solar radiation pressure acceleration of a GPS satellite and the claimed accuracy of 3% for the T20 model, the expected error in acceleration is approximately $3 \cdot 10^{-9} m/s^2$. Furthermore the size of the Y-bias, which is not included in the ROCK models, is about $1 \cdot 10^{-9} m/s^2$. The effect of both error sources is about 3 meters (RMS) over 24-hours! Of course, we have to keep in mind that the ROCK models were developed for orbit estimation using pseudo-range data. With pseudo-ranges, position estimates with an accuracy of about 1 meter may be obtained. For this type of accuracy the ROCK models are adequate to serve as (a priori) model, provided the scale term and the Y-bias are estimated.

The ROCK models are inadequate for IGS-type accuracies, e.g., centimeter-type orbit accuracies, even if the scale and Y-bias are estimated. This is also obvious from the additional orbit and solar radiation pressure parameters which most of the IGS ACs are estimating, be they deterministic and/or stochastic in nature. However, additional orbit parameters may weaken the GPS solutions significantly, especially the LOD (length of day) estimates. Therefore it would be advantageous if an improved RPR model could be developed. The experiences gained from our

IGS analysis efforts in recent years have indicated that it should indeed be possible to derive an improved RPR model [*Springer* et al., 1998].

Below we first present a summary of the most recent orbit results. Because these results are based on the extended CODE orbit model (ECOM) [*Beutler* et al., 1994], a short description of the ECOM is included as well. Secondly, results from orbit estimation using the GPS (IGS) precise orbits as pseudo-observations (orbit fit) are presented. Based on these results an ''optimal'' orbit parameterization is proposed. Using this parameterization and all CODE final orbits (and EOPs), as submitted to the IGS, a long time series for the selected (optimal) set of RPR parameters is generated. Only the results after January 1, 1996 are used to generate our new solar radiation pressure model. The article concludes with an evaluation of the quality of our RPR model and a short summary.

The Extended CODE Orbit Model

In *Beutler* et al. [1994] our orbit model, the extended CODE orbit model (ECOM), is presented and discussed in detail. We only summarize its basic characteristics. The considerations behind the ECOM are similar to those underlying the Colombo model [*Colombo*, 1989]. The principal difference resides in the fact that the ECOM considers the Sun as the major ''error source'' for the orbits, whereas the gravity field of the Earth plays this role in the Colombo model. The Colombo model uses the radial, along-, and cross-track directions as the three orthogonal directions, whereas the D-, Y-, and B-directions are used by the ECOM. The D-direction is the direction satellite-Sun, positive towards the Sun. The Y-direction is the same as in the case of the ROCK models pointing along one of the solar panel axis. The B-direction completes the right handed system, Notice that in earlier publication the B-axis was referred to as X-axis. To avoid confusion with the X-axis of the ROCK models, the B-axis is introduced here to designate the third axis of the ECOM. *Beutler* et al. [1994] demonstrated that the performance of the ECOM is superior to that of the Colombo model, which is a clear indication that solar radiation pressure is indeed the major error source in the GPS satellite orbit model. In the ECOM the acceleration $\vec{a}_{RPR}$ due to the solar radiation pressure is written as:

$$\vec{a}_{RPR} = \vec{a}_{ROCK} + D(u) \cdot \vec{e}_D + Y(u) \cdot \vec{e}_Y + B(u) \cdot \vec{e}_B \qquad (1)$$

where $\vec{a}_{ROCK}$ is the acceleration due to the ROCK model, and

$$\begin{aligned} D(u) &= D + D_{Cu} \cdot \cos u + D_{Su} \cdot \sin u \\ Y(u) &= Y + Y_{Cu} \cdot \cos u + Y_{Su} \cdot \sin u \\ B(u) &= B + B_{Cu} \cdot \cos u + B_{Su} \cdot \sin u \end{aligned} \qquad (2)$$

where D, D_{Cu}, D_{Su}, Y, Y_{Cu}, Y_{Su}, B, B_{Cu}, and B_{Su} are the nine parameters of the ECOM, and

$\vec{e}_D$ is the unit vector satellite-Sun, positive towards to Sun,
$\vec{e}_Y$ is the unit vector along the spacecraft's solar-panel axis (*Fliegel* et al. [1985])
$\vec{e}_B$ $\vec{e}_D \times \vec{e}_Y$, and
u is the argument of latitude of the satellite.

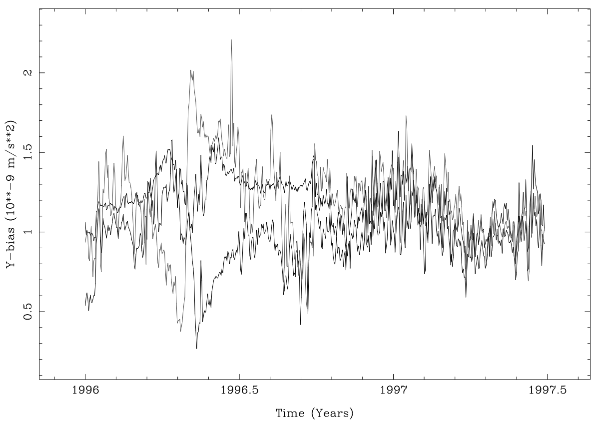
The ECOM is a generalization of the standard orbit model which uses only two parameters to account for the solar radiation pressure, namely D and Y. Note that the Y-direction of the ECOM corresponds to the Y-direction of the body-fixed coordinate system. Although not really a solar radiation pressure model in the sense of the ROCK models, the ECOM does consider solar radiation pressure to be the major perturbing force acting on the GPS satellites. Therefore the ECOM provides an excellent tool to study the effects of the solar radiation pressure on the GPS satellites. It allows to detect in which direction the most significant unmodeled RPR forces act on the GPS satellites.

There are two methods to study the effects of different parameters of the ECOM on the orbit estimates. The first, and most reliable method is to use the ECOM in real orbit estimation procedures using GPS observations, very much like the routine orbit estimation performed at CODE as part of its IGS activities. The second method is to use the orbital positions as provided by CODE, in the precise orbit format (SP3), as ''pseudo-observations'' estimating an arc extending over several days which gives the best fit, in a least squares sense, to the observations. The second method is less correct but computationally much more efficient than the first. The generation of a 3-day arc using the ''orbit fit method'' typically takes 1 minute whereas the ''orbit estimation'' method will take several hours. Results from both methods, orbit estimation and orbit fit, will be discussed in the following sections.

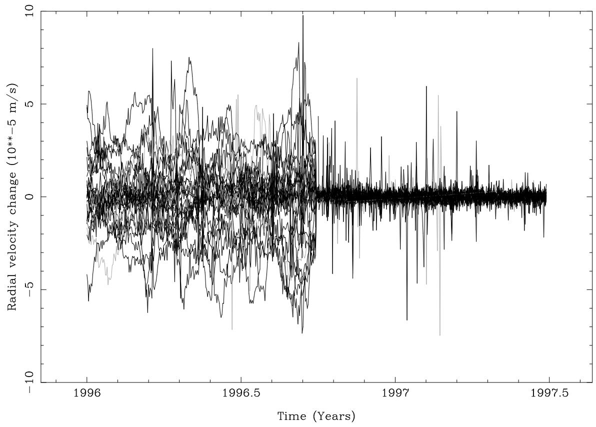
Orbit estimation using GPS observations

In 1996 the ECOM was fully implemented into the Bernese GPS Software [*Rothacher and Mervart*, 1996]. It was expected that not all nine parameters of the ECOM could (and should) be estimated when estimating 3-day arcs from real GPS data. Initial tests [*Springer* et al., 1996] indicated that it is best not to solve for ''B-terms'', but to estimate the constant and periodic terms in the D- and Y-directions plus small velocity changes (pseudo-stochastic pulses) in the radial and along-track directions. A careful analysis of this parameterization showed that it implied a significant degradation of the quality of the LOD estimates. It was therefore decided to systematically test different parameters of the ECOM in order to find the ''optimal'' parameterization. In *Springer* et al. [1998] a detailed description of results from two extensive tests using the ECOM is given. The difference between the two extensive test series is, that in the first test series pseudo-stochastic pulses [*Beutler* et al., 1996] were always estimated (stochastic test series) whereas in the second test series (deterministic test series) they were not estimated.

The stochastic test series showed that the estimation of the constant and periodic terms in the B-direction (B, B_{Cu}, and B_{Su}), in addition to the estimation of the constant terms in the D- and Y-direction, and the pseudo-stochastic pulses in the radial and along-track directions, significantly improves the quality of the orbit estimates. An improvement was seen in all estimated parameters: orbits, station coordinates, and Earth orientation parameters (EOPs). Only for the LOD estimates a small, but significant, degradation in quality was observed. The improvement of the orbit quality was estimated to be a factor of two to three, compared to the results without estimating the three terms in the B-direction! As a direct consequence of these tests the estimation of the B-terms was implemented for the generation of the CODE contributions to the IGS on September 29, 1996.



(a) Y-bias estimates for the satellites in orbital plane A (PRNs 9, 25, 27)



(b) Estimated radial pseudo-stochastic pulses for all satellites except PRN 23

Figure 1: Estimated Y-bias (Y) and radial pseudo-stochastic pulses using the two different CODE orbit parameterizations. The change of the orbit parameterization in September 1996 is clearly visible.

The second, deterministic, test series confirmed that the periodic terms in the B-direction most significantly reduce the orbit model deficiencies. Evidence was presented that the periodic signals in the Y-direction reduce the orbit model deficiencies as well. The periodic signals in the B-direction, however, were shown to be more important than those in the Y-direction. The deterministic test series further showed that a purely deterministic orbit parameterization, consisting of the constant terms in the D- and Y-directions plus periodic terms in the D- and B-directions, gives excellent orbit results. Because of a degradation of the LOD estimates this deterministic orbit model is currently not used for the IGS activities at CODE.

The results based on the CODE routine orbit estimates using the standard (R3) and new (X3) orbit parameterizations (3 B-terms) showed that the behaviour of the estimated stochastic and deterministic orbit parameters significantly improves with the new orbit parameterization. This is true in particular for the Y-bias (see Figure 1(a)), and the radial pseudo-stochastic pulses (see Figure 1(b)). Tests without estimating the radial pseudo-stochastic pulses showed that with the new orbit parameterization these pulses no longer have to be estimated. Considering the fact that pseudo-stochastic pulses are meant to absorb orbit model deficiencies it is clear that the modeling deficits are significantly reduced in the new orbit parameterization. Because the behaviour of all estimated RPR parameters in the X3 solutions is ‘‘predictable’’ it is expected that an improved RPR model may be developed.

Orbit estimation using satellite positions as pseudo-observations

The essential difference between orbit estimation, using real (GPS) data, and orbit fitting, using previously determined satellite positions as pseudo-observations lies in the type of observations used. For the orbit estimation double-difference GPS carrier phase observations are used whereas for the orbit fit procedure the observations are position vectors of the satellites.

The goal of the orbit fit tests is to find the optimal orbit parameterization. For this purpose a ''standard test'' was developed to compare the results. This standard test consists of a 7-day orbit fit using the CODE final products (i.e. precise orbits plus their respective EOPs). The resulting 7-day arc is extrapolated, using orbit integration, for 48 hours. The last 24 hours of this orbit prediction are compared to the CODE final orbit for the same day. The period from March 13 to March 21 in 1997 was selected as test interval. The following quantities are considered as quality indicators for the orbit parameterization:

- the RMS of the 7-day fit ,
- the RMS of comparison of predicted and ''true'' orbits, and
- the median of this comparison.

First, we verified that our standard orbit fit test gave similar results as the deterministic orbit estimation test discussed earlier. Only one small anomaly was detected in the estimation of the periodic terms in the Y- and B-direction. In the orbit fit the effect of the periodic terms in these two directions are almost identical whereas in the orbit estimation test a significant difference was observed speaking in favor of the periodic terms in the B-direction.

Software modifications were made to allow for periodic terms in two other coordinate systems: the satellite-fixed reference frame (X, Y, Z) and the ''classical'' orbit system (radial, along-, and cross-track (R, S, W)). In addition, the argument for the periodic terms was slightly changed to account for the position of the Sun with respect to the ascending node. This change is a consequence of the assumption that the solar radiation pressure is the major ''error source'' in the GPS orbit modeling. It is therefore logical to relate the time argument of the periodic signals to the position of the Sun in the orbital plane, [*Rothacher* et al., 1995]. Thus, the argument of latitude is corrected for the argument of latitude of the Sun in the orbital plane (u_0) (see Figure 2)

After extensive tests, using many different combinations of parameters, a small set of optimal orbit parameterizations was found. All candidate parameterizations were subsequently used in real orbit estimation using one full week of GPS data. This test confirmed that all 5 parameterizations perform very well apart from some correlation with the LOD. The choice of the ''optimal'' parameterization was based on a slightly better performance and its resemblance to the ROCK model. This ''optimal'' parameterization, consisting of six parameters, three constants and three periodic terms, describes the acceleration $\vec{a}_{RPR}$ due to the solar radiation pressure in the following way:

$$\begin{aligned} \vec{a}_{RPR} &= D \cdot \vec{e}_D + Y \cdot \vec{e}_Y + B \cdot \vec{e}_B + Z_1 \sin(u - u_0) \cdot \vec{e}_Z + \\ & \quad \{X_1 \sin(u - u_0) + X_3 \sin(3u - u_0)\} \cdot \vec{e}_X \end{aligned} \tag{3}$$

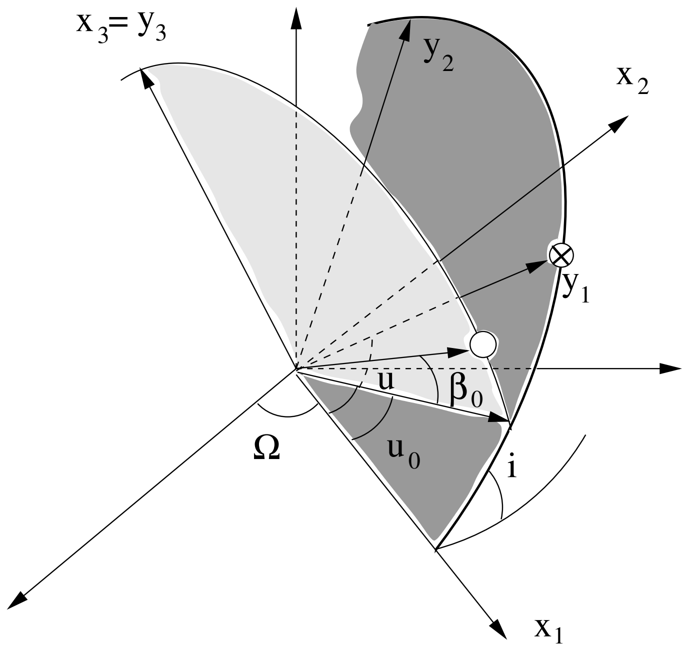


Figure 2: Definition of the latitude of the Sun in the orbital plane (u_0) and the elevation of the Sun above the orbital plane (β_0).

where,

- $\vec{e}_X$ the satellite-fixed X-axis, positive toward the half plane that contains the Sun,
- $\vec{e}_Y$ the satellite-fixed Y-axis, pointing along the spacecraft's solar-panel axis ($\vec{e}_Z \times \vec{e}_X$),
- $\vec{e}_Z$ the satellite-fixed Z-axis, the vector satellite-Earth, positive towards the Earth,
- u_0 is the argument of latitude of the Sun in the orbital plane, see Figure 2,

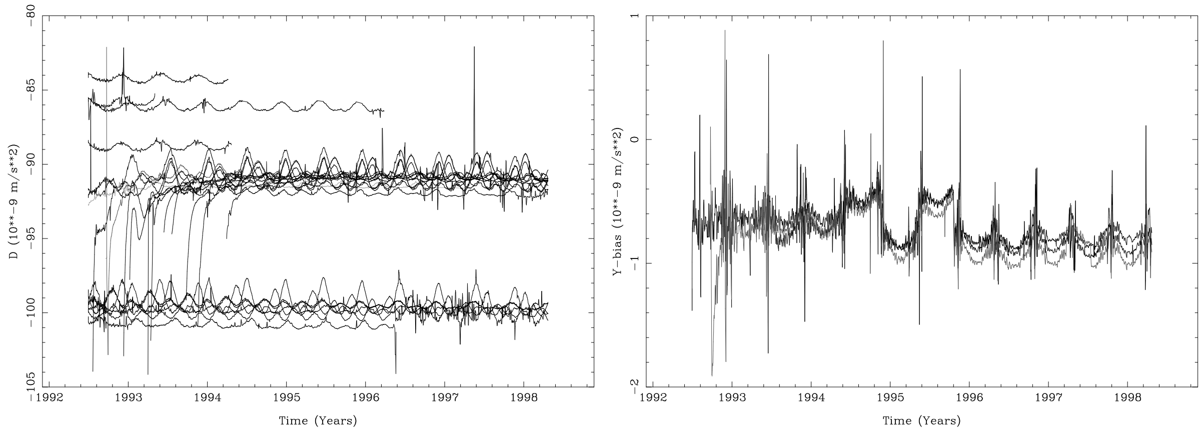
and D, Y, and B are the three constant terms and, Z_1, X_1, and X_3 are the three periodic terms of the CODE RPR model. Note that D, Y and B are related to the ECOM whereas Z_1, X_1, and X_3 are new parameters. Z_1 and X_1 have a similar behavior as the periodic terms in B and D, respectively.

The New Solar Radiation Pressure Model

Using the ''optimal'' orbit parameterization (3) all final CODE orbits with their respective EOPs, as submitted to the IGS since June 1992, were used in an orbit fit procedure. An arc length of 5 days was chosen and **no** a priori solar radiation pressure model was used. This resulted in a long time series, covering 5.5 years, of estimates for the selected (optimal) set of six RPR parameters. It was hoped that, after careful analysis, this time series may be used to derive a new solar radiation pressure model. Figure 3 shows the estimated values for the direct solar radiation pressure (D in (3)) and the estimated values for for the Y-bias (Y in (3)) accelerations as function of time over the full 5.5 years. Jumps are visible in the Y-bias time series. These jumps are related to the ''bias'' changes in the attitude system of the GPS satellites. These biases have been kept constant since approximately November 1995. Futhermore, the eclipse phases can clearly be seen in the Y-bias estimates: the estimates are somewhat anomalous during these phases.

A careful analysis of the estimated parameters as a function of time showed that the behaviour of satellites within one orbital plane is very similar. Clear annual and semiannual signals are present. Assuming that the Sun causes the observed signals it is logical to study the behaviour of the RPR parameters as function of the angle of the Sun above the orbital plane (angle β_0), see Figure 2. Note that the satellite is in eclipse if $|\beta_0| \leq 14^o$. In Figure 4 the same two time series for the direct solar radiation pressure and Y-bias accelerations are shown but now as function of the angle β_0 of the Sun above the orbital plane. For the Y-bias a shorter time interval was selected to exclude the observed jumps, but more satellites are shown.

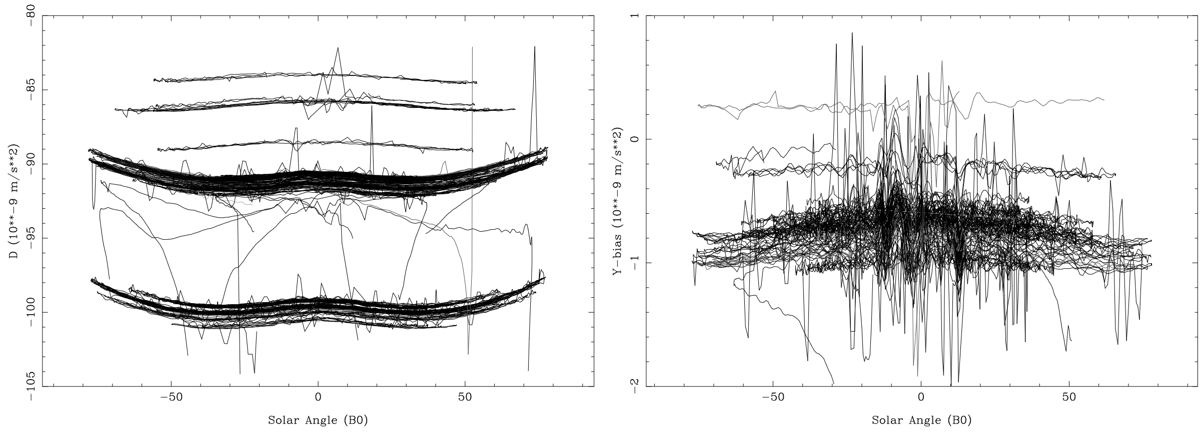
The estimates of the direct solar radiation pressure acceleration and the Y-bias acceleration are very similar for all (Block II and IIa) satellites indicating that a model can be derived for these parameters. The same is true for the constant term in the B-direction and for the once per revolution periodic term in the Z-direction. Both periodic signals in the X-direction do not show a very clear signal. Nevertheless, a model was estimated for these parameters as well. Block I satellites, the uppermost lines in Figures 3a and 4a, show a slightly different characteristic. Although their behaviour is also predictable, no attempt was made to create a model for the Block I satellites. Due to the ''jumps'' in the estimates of the Y-bias, the model for this parameter had to be based on the estimates since 1996 only. It turned out that the performance of the complete model was better if all model parameters were uniquely based on the most resent results (since



(a) Direct solar radiation pressure acceleration for the Block I, II, and IIa satellites

(b) Y-bias acceleration for the satellites in orbital plane A (PRNs 9, 25, 27)

Figure 3: Estimated direct solar radiation pressure acceleration (D) and Y-bias (Y) acceleration as function of time over the interval from June 1992 to April 1998



(a) Direct solar radiation pressure acceleration for the Block I, II, and IIa satellites

(b) Y-bias acceleration for the Block II and IIa satellites

Figure 4: Estimated direct solar radiation pressure acceleration (D) and Y-bias (Y) acceleration as function of the angle β_0 of the Sun above to the orbital plane. For D the complete interval from June 1992 to April 1998 is shown whereas for Y only the last part since 1996 is included.

1996). Apparently the (significant) modeling improvements made over the last few years are important when deriving a solar radiation pressure model.

The time series, since 1996, were used to derive a β_0-dependent function for each of the six parameters. The functions for the six parameters, giving the best fit through the time series, are:

$$\boxed{\begin{aligned}
D(\beta_0) &= D_0 + D_{C2}\cos(2\beta_0) + D_{C4}\cos(4\beta_0) \\
Y(\beta_0) &= Y_0 + Y_C\cos(2\beta_0) \\
B(\beta_0) &= B_0 + B_C\cos(2\beta_0) \\
Z_1(\beta_0) &= Z_0 + Z_{C2}\cos(2\beta_0) + Z_{S2}\sin(2\beta_0) + Z_{C4}\cos(4\beta_0) + Z_{S4}\sin(4\beta_0) \\
X_1(\beta_0) &= X_{10} + X_{1C}\cos(2\beta_0) + X_{1S}\sin(2\beta_0) \\
X_3(\beta_0) &= X_{30} + X_{3C}\cos(2\beta_0) + X_{3S}\sin(2\beta_0)
\end{aligned}} \tag{4}$$

where,

β_0 is the elevation of the Sun above the orbital plane, see Figure 2.

Using (3) and (4) the CODE RPR model may now be written as:

$$\begin{aligned}
\vec{a}_{RPR} = \; & D(\beta_0)\cdot\vec{e}_D + Y(\beta_0)\cdot\vec{e}_Y + B(\beta_0)\cdot\vec{e}_B + Z_1(\beta_0)\sin(u-u_0)\cdot\vec{e}_Z + \\
& \{X_1(\beta_0)\sin(u-u_0) + X_3(\beta_0)\sin(3u-u_0)\}\cdot\vec{e}_X
\end{aligned} \tag{5}$$

Note that all three constants ($D_0,\ Y_0,\ B_0$) were chosen to be satellite-specific and that the Z_0-term was chosen to be Block-type-dependent. The values for all parameters are given in the Appendix. Note that the model is only valid for Block II and IIa satellites. Furthermore, the values given for PRN8 should be used with care because this satellite was launched late in 1997 and by the end of the year was still in its ‘‘outgassing’’ phase. Satellite PRN23 should be used with care as well due to the problems with the orientation of its solar panels. The results indicate, however, that it should be possible to derive a tailored RPR model for PRN23.

Evaluation of the New Solar Radiation Pressure Model

Four different tests were made to evaluate the new solar radiation pressure model.

- The effect of the parameters of our RPR model on the satellite positions was determined to get an idea of the significance of the individual terms of the model.
- An error budget of the model was derived, based on the residuals of the RPR series to get an idea of the remaining model errors.
- The model was compared, using our standard test, to other RPR models.
- The model was used in a real parameter estimation using one full week of GPS observations.

The effects of the different parameters of the new RPR model on the orbit were estimated by integrating a given set of osculating Keplerian elements over a time period of one day (24 hours), once with the parameter turned on and once with the parameter turned off. The RMS of the difference between the two resulting orbits, over the full 24 hour period, was then computed to get an idea of the size of the effect. The results are given in Table 2.

Table 2: Effect of the individual parameters of the new RPR model on the GPS satellite orbits over 24 hours.

Parameter	Effect (cm)			
	Radial	Along	Cross	Total
$D(\beta_0)$	2900	8700	300	9200
$Y(\beta_0)$	49	350	8	354
$B(\beta_0)$	2	29	3	29
$Z_1(\beta_0)$	15	32	0	36
$X_1(\beta_0)$ and $X_3(\beta_0)$	2	11	0	11

As expected the $D(\beta_0)$ (direct solar radiation term) and $Y(\beta_0)$ (Y-bias) give the largest contributions. However, the contributions of the $B(\beta_0)$-term and the periodic term in the Z-direction (radial direction!) are not negligible! Note that the periodic Z-term has a signature very similar to the periodic terms in the B-direction, which CODE uses when computing its IGS orbit products. The periodic terms in the X-direction have an effect of only 11 cm. The typical RMS of the 5-day fits, used for the model development, is at the level of 5 cm. The 11 cm effect is thus close to the noise level of the solutions. However, the IGS orbit combinations show an orbit consistency of about 4 cm between the orbits of different ACs; an 11 cm effect might be significant.

Our RPR model is based on a time series of parameter estimates based on 5-day arcs using the ''final'' orbit positions from CODE. The RMS of the parameter estimation, after subtracting the estimated RPR model, is used to estimate the remaining errors in the model. For this purpose the RMS value was introduced as a ''bias'' in the corresponding RPR parameter and a 24-hour orbit integration was performed with this bias included. The difference between the biased orbit and the original orbit are a measure of the remaining orbit model errors. The total error budget was estimated by introducing the RMS value for all parameters as bias. Surprisingly enough the largest error source stems from the two periodic terms in the X-direction. This is remarkable in view of the very small effect these parameters have on the orbit. The estimated errors from the other parameters are all below the 20 cm level. The total error budget is estimated to be about 80 cm. To verify this number, our standard test was used without estimating any parameters (except the 6 osculating Keplerian elements). The RMS of this 7-day fit should then be comparable to the estimated model error. The result was comparable, 52 cm 7-day orbit fit, but the error budget (80 cm) seems to be somewhat pessimistic. This may be caused by the relatively large error of the X-periodic terms. Also, the arc length of our standard test (7 days) is longer than the arc length used for the RPR parameter estimates (5 days). Therefore the remaining orbit model error is estimated to be of the order of 50 cm only!

The JPL IGS analysis center has successfully developed a new RPR model as well [*Bar-Sever*, 1997]. To test the performance of different RPR models our standard test was used once more. Table 3 gives the results of the standard test using the RPR models available: ROCK, JPL, and CODE. It shows the RMS of fit using 7 days of precise orbits and the RMS and median of the residuals of the prediction comparison. Again the CODE final orbit positions and EOPs were used as pseudo observations. In all cases only the scale term (or a constant acceleration in the direction sun-satellite (D in (3)) and the Y-bias (Y in (3)) were estimated. Only for the solution labelled ''BEST'' all 9 parameters of the ECOM(2) were estimated. This solution is given as a reference. Furthermore, the ROCK model was used in two different ways. First, it was used as a priori model and the accelerations D and Y were estimated on top of the model (model: T20), which represents the way the ROCK model is normally used in the Bernese GPS Software. Secondly, it was used by estimating a scale factor for the complete ROCK model and the acceleration in the Y-direction (model: T20 scaled), which represents the recommended usage of the ROCK model.

Table 3: Orbit Fit (7 days) and orbit extrapolation (2 days) using different RPR models. Only scale (or D) and Y-bias estimated.

RPR-MODEL	RMS of FIT (cm)	Prediction Median (cm)	Prediction RMS (cm)
No Model	75	133	159
T20	76	134	161
T20 Scaled	72	119	151
JPL Scaled	10	45	58
CODE	6	17	31
''BEST'' (9 RPR par.)	5	17	22

Table 3 shows that including the ROCK model as a priori RPR model hardly gives an improvement, both in the fit and in the prediction, compared to not including an a priori model. Although it was clear for a long time that the ROCK models are not appropriate for precise geodetic work, the clear result is is a surprise! Both, the CODE and JPL RPR models, perform much better than the ROCK model. The results of the CODE model are close to the ''best possible'' results. This means that the reduction of the number of estimated RPR parameters (from 9 to 2), does not significantly degrade the accuracy of the results. This reduction of parameters should make GPS orbit predictions more reliable. This is important because integrity of the predicted orbits is the most crucial factor for real time GPS data analysis [*Martin Mur* et al., 1998].

Finally, the new RPR model was tested in a real GPS data processing experiment using one full week of data (7 days of 3-day solutions). Four different solutions were generated. For the first two solutions our standard $(D,\ Y)$ and new $(D,\ Y,\ B,\ B_{Cu},\ B_{Su})$ orbit solutions were generated using the ROCK model as a priori model. For the second set of two solutions the results were generated using the new CODE RPR model as a priori model. A significant improvement was observed for the standard solution $(D,\ Y)$. In fact, the standard solution using the CODE model has become almost as good as the two X3-type $(D,\ Y,\ B,\ B_{Cu},\ B_{Su})$ solutions, which gave practically identical results. This is an important result because three orbit parameters (the three B-terms) become obsolete! The slight difference observed in quality is most likely caused by the eclipsing satellites, which are not treated in any special way in the CODE model. It is known from *Bar-Sever* [1995] that eclipsing satellites should be treated separately.

Summary and Outlook

It has been shown that the new solar radiation pressure model as developed by CODE is superior to the ROCK model by almost one order of magnitude. The remaining model error was estimated to be about 50 cm, whereas for the ROCK model the error is about 300 cm. Although a significant improvement could be achieved with the new RPR model, it should be considered as a ‘‘first attempt’’ only. In the near future more time and effort will (have to) be spent on RPR models. Different models are required for the Block I and IIr satellites and also for PRN23. Furthermore, the behaviour of some of the parameters of the RPR model is anomalous during the eclipse phases. Special eclipse models should be derived.

The implementation of the CODE RPR model will improve the quality of orbit estimates. The number of required (orbit) parameters is reduced, which will strengthen the GPS solutions significantly. The generation of so-called ‘‘rapid’’ products may profit from this development in particular. The quality of predicted orbits may also improve.

The new RPR model allows it to generate GPS orbits based on SLR observations only. So far, the limited number of SLR observations and the large number of required orbit parameters made it almost impossible to generate accurate GPS orbits based on SLR data only. With our new RPR model we are in a much better position because it allows a 7-day fit at the 6 cm level solving for only two parameters.

References

Bar-Sever, Yoaz E. (1995), A new Model for GPS Yaw Attitude, in *Special Topics and New Directions*, edited by G Gendt and G. Dick, GeoForschungsZentrum, Potsdam, Germany, May 15--18 1995.

Bar-Sever, Yoaz E. (1997), New and Improved Solar Radiation Models for GPS Satellites Based on Flight Data, Final Report fo Air Force Materiel Command Space and Missile Systems Center/CZSF, April 12 1997.

Beutler, G., E. Brockmann, W. Gurtner, U. Hugentobler, L. Mervart, and M. Rothacher (1994), Extended Orbit Modeling Techniques at the CODE Processing Center of the International GPS Service for Geodynamics (IGS): Theory and Initial Results, *Manuscripta Geodaetica*, *19*, 367--386, April 1994.

Beutler, G., E. Brockmann, U. Hugentobler, L. Mervart, M. Rothacher, and R. Weber (1996), Combining Consecutive Short Arcs into Long Arcs for Precise and Efficient GPS Orbit Determination, *Journal of Geodesy*, *70*, 287--299.

Colombo, O. L. (1989), The Dynamics of Global Positioning Orbits and the Determination of Precise Ephemerides, *Journal of Geophysical Research*, *94*(B7), 9167--9182.

Fliegel, H. F., and T. E. Gallini (1989), Radiation Pressure Models for Block II GPS Satellites, in *Proceedings of the Fifth International Symposium on Precise Positioning with the Global Positioning System*, pp. 789--798, National Geodetic Survey, NOAA, Rockville, Md., April 1989.

Fliegel, H. F., W. A. Feess, W. C. Layton, and N. W. Rhodus (1985), The GPS Radiation Force Model, in *Proceedings of the First International Symposium on Precise Positioning with the Global Positioning System*, edited by Clyde Goad, pp. 113--119, National Geodetic Survey, NOAA, Rockville, Md., March 1985.

Fliegel, H. F., T. E. Gallini, and E. R. Swift (1992), Global Positioning System Radiation Force Model for Geodetic Applications, *Geophysical Research Letters*, *97*(B1), 559--568.

Martin Mur, T.J., T.A. Springer, and Y. Bar-Sever (1998), Orbit Predictions and Rapid Products, in *Proceedings of the IGS Analysis Center Workshop*, ESA, Darmstadt, Germany, February 1998, This Volume.

Rothacher, M., G. Beutler, and L. Mervart (1995), The Perturbation of the Orbital Elements of GPS Satellites Through Direct Radiation Pressure, in *IGS Workshop Proceedings on Special Topics and New Directions*, edited by G. Gendt and G. Dick, pp. 152--166, GeoForschungsZentrum, Potsdam, Germany, May 15--18 1995.

Rothacher, Markus, and Leos Mervart (1996), *The Bernese GPS Software Version 4.0*, Astronomical Institute, University of Berne, September 1996.

Springer, T. A., M. Rothacher, and G. Beutler (1996), Using the Extended CODE Orbit Model: First Experiences, in *IGS 1996 Analysis Center Workshop*, edited by R.E. Neilan *et al.*, pp. 13--25, Central Bureau, JPL, Pasadena, Ca, March 19--21 1996.

Springer, T. A., G. Beutler, and M. Rothacher (1998), Improving the Orbit Estimates of the GPS Satellites, *Journal of Geodesy*, January 1998, Accepted for publication.

Appendix: The CODE solar radiation pressure model

Table 4: Parameter values of the CODE solar radiation pressure model. The values for PRN8 and PRN13 should be used with care. PRN8 was launched by the end of 1997 end was still in its ‘‘outgassing’’ phase. PRN13 was also launched by the end of 1997 and is a new type of satellite (Block IIr) which most likely will have a different solar radiation pressure model.

PRN	Block	D_0 ($10^{-9}m/s^2$)	Y_0 ($10^{-9}m/s^2$)	B_0 ($10^{-9}m/s^2$)
2	II	-99.373	-0.6362	-0.0480
14	II	-99.290	-0.9064	0.2510
15	II	-98.985	-0.7048	0.4749
16	II	-99.108	-0.6496	0.1170
17	II	-99.010	-0.6604	0.0770
18	II	-99.359	-0.8683	0.4783
19	II	-99.850	-0.7057	0.1449
20	II	-100.396	-0.6642	0.4997
21	II	-99.477	-0.2592	-0.0996
1	IIa	-91.088	-0.7458	0.4868
3	IIa	-90.395	-0.5637	0.3960
4	IIa	-90.502	-0.7856	0.2487
5	IIa	-90.414	-0.7612	0.2309
6	IIa	-90.354	-0.7589	0.3092
7	IIa	-90.238	-1.0376	0.2241
8	IIa	-93.342	-1.8394	0.7143
9	IIa	-90.317	-0.7955	0.3569
10	IIa	-89.546	-0.7819	0.1772
22	IIa	-90.944	-0.7319	0.0179
23	IIa	-78.592	-0.7440	1.0843
24	IIa	-91.436	-1.0537	0.2214
25	IIa	-90.785	-0.8556	0.3851
26	IIa	-90.377	-0.9750	0.4144
27	IIa	-90.291	-0.9482	0.4224
28	IIa	-90.951	-0.8210	0.1303
29	IIa	-91.015	-0.9078	0.5188
30	IIa	-90.455	-0.8285	0.5409
31	IIa	-90.370	-0.6269	0.6173
13	IIr	-99.599	0.2801	1.6732

Table 5: The values of the CODE solar radiation pressure model

Parameters	Estimate ($10^{-9} m/s^2$)	Formal Error ($10^{-11} m/s^2$)
D_{C2}	-0.813	0.176
D_{C4}	0.517	0.124
Y_C	0.067	0.104
B_C	-0.385	0.572
Z_0 Block II	1.024	0.299
Z_0 Block IIa	0.979	0.184
Z_{C2}	0.519	0.248
Z_{S2}	0.125	0.149
Z_{C4}	0.047	0.261
Z_{S4}	-0.045	0.164
X_{10}	-0.015	0.157
X_{1C}	-0.018	0.297
X_{1S}	-0.033	0.168
X_{30}	0.004	1.655
X_{3C}	-0.046	3.118
X_{3S}	-0.398	1.773